ELEMENTS

OF

HYDRAULICS

A TEXT-BOOK

FOR SECONDARY TECHNICAL SCHOOLS

BY

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PREFACE

In the following pages the attempt is made to give a presentation of the subject of hydraulics without the use of higher mathematics. The degree of preparation required of the reader is merely that now given in high schools, and includes only arithmetic, algebra, trigonometry, and an elementary course in mechanics. In particular the author has had in mind the students in the upper classes of secondary technical schools, and it has been his aim to present the subject in such a manner that it may be readily comprehended by them.

It is believed that the essential principles and methods of hydraulics have been covered, although an expert can easily criticise the book on the ground that certain topics have been inadequately treated or omitted altogether. While it has been harder for the author to decide what should be omitted than what should be included, it has been his intention to discuss, as fully as seemed consistent with the assigned limit of space, those topics which are of greatest importance in practical engineering work.

MANSFIELD MERRIMAN.

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ELEMENTS OF HYDRAULICS

CHAPTER 1

HYDROSTATICS

ARTICLE 1. UNITS OF MEASURE

Hydraulics is that science which treats of water in motion. Hydrostatics is that part of hydraulics which treats of the equilibrium and pressure of water for the case when it is at rest. The unit of linear measure used in this book is the foot, while inches must always be reduced to feet for use in the hydraulic formulas. The units of volume are the cubic foot and the gallon, but the latter must always be reduced to cubic feet for insertion in the formulas. The gallon used in the United States contains 231 cubic inches, while in Great Britain the Imperial gallon is employed which is about 20 percent larger.

The unit of force is the pound, or the force exerted by gravity at the surface of the earth on a mass of matter called the avoirdupois pound. This unit is also used in measuring weights and pressures of water. The intensity of pressure is usually measured in pounds per square inch. The unit of time to be used in all hydraulic formulas is the second, although in numerical problems the time is often stated in minutes, hours, or days. Velocity is defined as the space passed over by a body in one second under the condition of uniform motion, so that velocities are to be always expressed in feet per second, or are to be reduced to these units if stated in miles per hour or

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otherwise. Acceleration is the velocity gained in one second, and it is measured in feet per second per second.

The unit of work is the foot-pound; that is, one pound lifted through a vertical distance of one foot. Energy is work which can be done; for example, a moving body has the ability to do a certain amount of work by virtue of its quantity of matter and its velocity, and this is called kinetic energy. Again, water at the top of a fall has the ability to do a certain amount of work by virtue of its quantity and its height above the foot of the fall, and this is called potential energy. Potential energy changes into kinetic energy as the water falls, and kinetic energy is either changed into heat or may be transformed, by means of a water motor, into useful work. Power is work done, or energy capable of being transformed into work, in a specified time, and the unit for its measure is the horse-power, which is 550 foot-pounds per second.

√ Prob. 1 *A*. When 3200 pounds of water fall every second from a height of 22 feet, what is the greatest horse-power that can be developed by the use of the falling stream?

√ Prob. 1 *B*. When one cubic foot of water, weighing 62½ pounds, falls each second from a vertical height of 11 feet, what horse-power can be developed by a motor which utilizes 80 percent of the energy?

ART. 2. PHYSICAL PROPERTIES OF WATER

At ordinary temperatures pure water is a colorless liquid which possesses almost perfect fluidity; that is, its particles have the capacity of moving over each other, so that the slightest disturbance of equilibrium causes a flow. It is a consequence of this property that the surface of still water is always level; if several vessels or tubes be connected, as in Fig. 1, and water be poured into one of them, it rises in the others until the free surfaces are in the same level plane.

The free surface of water is in a different molecular condition from the other portions, its particles being drawn together by stronger attractive forces, so as to form what may be called the "skin of the water," upon

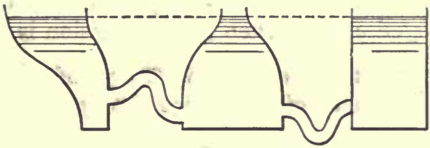


Fig. 1

which insects may walk or a needle be caused to float. The skin is not immediately pierced by a sharp point which moves slowly upward toward it, but a slight elevation occurs, and this property enables precise determinations of the level of still water to be made by the hook gage (Art. 53).

At about 32° Fahrenheit a great alteration in the molecular constitution of water occurs, and ice is formed. If water be kept in a perfectly quiet condition, it is found that its temperature can be reduced to 20° or even to 15° Fahrenheit, before congelation takes place, but at the moment when this occurs the temperature rises to 32°. The freezing-point is hence not constant, but the melting-point of ice is always at the same temperature of 32° Fahrenheit or 0° centigrade. While water freezes at 32° Fahrenheit, yet its maximum density is reached at 39°.3 Fahrenheit. At this latter temperature its specific gravity is 1.0 while at 32° it is 0.99987. As the temperature rises above that of maximum density the specific gravity of water steadily grows smaller until the boiling-point is reached at 212° Fahrenheit when it is 0.95865.

Prob. 2. If a cubic foot of pure water weighs 62.424 pounds at the temperature of maximum density, what is the weight of a cubic foot at 32° F.?

ART. 3. THE WEIGHT OF WATER

The weight of water per unit of volume depends upon the temperature and upon its degree of purity. The following approximate values are, however, those generally employed except when great precision is required:

1 cubic foot of water weighs 62.5 pounds
1 U. S. gallon of water weighs 8.355 pounds

These values will be used in this book, unless otherwise stated, in the solution of the examples and problems.

The weight per unit of volume of pure distilled water is the greatest at the temperature of its maximum density, 39°.3 Fahrenheit, and least at the boiling-point. For ordinary computations the variation in weight due to temperature is not considered, but in tests of the efficiency of hydraulic motors and of pumps it should be regarded. The following are the weights of pure distilled water, in pounds per cubic foot, for a few temperatures in the Fahrenheit scale:

At 32°	62.42	At 80°	62.22
39.3	62.424	120	61.72
45	62.42	150	61.20
50	62.41	180	60.59
60	62.37	212	59.84

Waters of rivers, springs, and lakes hold in suspension and solution inorganic matters which cause the weight per unit of volume to be slightly greater than for pure water. River waters are usually between 62.3 and 62.6 pounds per cubic foot, depending upon the amount of impurities and on the temperature, while the water of some mineral springs has been found to be as high as 62.7. It appears that, in the absence of specific information regarding a particular water, the weight 62.5 pounds per cubic foot is a fair approximate value to use. It also has the advantage of being a convenient number in computations, for 62.5 pounds is 1000 ounces, or $^{1000}/_{16}$ is the equivalent of 62.5.

Brackish and salt waters are always heavier than fresh water. For the Gulf of Mexico the weight per cubic foot is about 63.9, for the oceans about 64.1, while for the Dead Sea there is stated the value 73 pounds per cubic foot. For Great Salt Lake the weight of water varies from 69 to 76 pounds per cubic foot. The sewage of American cities is impure water which weighs from 62.4 to 62.7 pounds per cubic foot, but the sewage of European cities is somewhat heavier on account of the smaller amount of water that is turned into the sewers.

Prob. 3. How many gallons of water are contained in a pipe 3 inches in diameter and 12 feet long? How many pounds of water are contained in a pipe 6 inches in diameter and 24 feet long?

ART. 4. ATMOSPHERIC PRESSURE

Torricelli in 1643 discovered that the atmospheric pressure would cause mercury to rise in a tube from which the air had been exhausted. This instrument is called the mercury barometer, and owing to the great density of mercury the height of the column required to balance the atmospheric pressure is only about 30 inches. When water is used in the vacuum tube, the height of the column is about 34 feet. In both cases the weight of the barometric column is equal to the weight of a column of air of the same cross-section as that of the tube, both columns being measured up from the common surface of contact.

The atmosphere exerts its pressure with varying intensity as indicated by the readings of the mercury barometer. At and near the sea level the average reading is 30 inches, and as mercury weighs 0.49 pounds per cubic inch at common temperatures, the average atmospheric pressure is 30×0.49 or 14.7 pounds per square inch. One atmosphere therefore exerts a pressure of 14.7 pounds per square inch. Then a pressure of two atmospheres is 29.4 pounds per square inch.

Pascal, in 1646, carried a mercury barometer to the top of a mountain and found that the height of the mercury column decreased as he ascended. It was thus definitely proved that the cause of the ascent of the liquid in the vacuum tube was due to the pressure of the air. Since mercury is 13.6 times heavier than water, a column of water should rise to a height of $30 \times 13.6 = 408$ inches $= 34$ feet under the pressure of one atmosphere, and this was also found to be the case. The following table shows heights of the mercury and water barometer with the corresponding pressures in pounds per square inch and in atmospheres. It also gives approximate elevations above sea level corresponding to barometer readings, provided the reading at sea level is 30 inches. In the last line are approximate boiling-points of water corresponding to the readings of the mercury barometer.

Mercury barometer in inches =	31	30	29	28	27	26	20
Water barometer in feet =	35.1	34.0	32.9	31.7	30.6	29.5	22.7
Pressure in pounds per square inch =	15.2	14.7	14.2	13.7	13.2	12.7	9.8
Pressure in atmospheres =	1.03	1.00	0.97	0.93	0.90	0.86	0.67
Elevations in feet =	−890	0	+920	+1880	+2870	+3900	+11050
Boiling point of water, Fahrenheit =	213°.9	212.2	210.4	208.7	206.9	205.0	192.4

The atmospheric pressure must be taken into account in many computations on the flow of water in tubes and pipes. It is this pressure that causes water to flow in siphons and to rise in tubes from which the air has been exhausted. By virtue of this pressure the suction pump is rendered possible, and all forms of injector pumps depend upon it to a certain degree.

Prob. 4 *A*. A mercury barometer reads 29 inches at the foot of a hill, and at the same time another barometer reads 28 inches at the top of the hill. What is the difference in height between the two stations?

Prob. 4 *B*. Find, from the above table, the approximate height above the sea level of a mountain on which water boils at a temperature of 206° F.

ART. 5. TRANSMISSION OF PRESSURE

One of the most remarkable properties of a fluid is its capacity of transmitting a pressure, applied at one point of the surface of a closed vessel, unchanged in intensity, in all directions, so that the effect of the applied pressure is to cause an equal force per square inch upon all parts of the enclosing surface. Pascal, in 1646, was the first to note that great forces could be produced in this manner; he saw that the total pressure increased proportionally with the area of the surface. Taking a closed barrel filled with water, he inserted a small vertical tube of considerable length tightly into it, and on filling the tube the barrel burst under the great pressure thus produced on its sides, although the weight of the water in the tube was quite

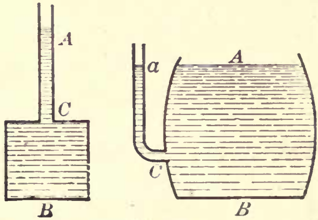


Fig. 2

small. The first diagram in Fig. 2 represents Pascal's barrel, and it is seen that the unit-pressure in the water at B is due to the head AB and independent of the size of the tube AC.

Bramah built the first successful hydraulic press in 1796. This machine has two pistons of different sizes, and a force applied to the small piston is transmitted through the fluid and produces an equal unit-pressure at every point on the large piston. The applied force is here multiplied to any required extent, but the work performed by the large piston cannot exceed that imparted to the fluid by the small one. Let a and A in Fig. 3 be the areas of the small and large pistons, and p the pressure in pounds per square unit applied to a; then the unit-pressure in the fluid is p, and the total pressure on the small piston is pa, while that on the large piston is pA. Let the distances through which the pistons move during one stroke be d and D. Then the imparted work is pad, and the performed work, neglecting frictional resistances, is pAD. Consequently $ad = AD$, and since a is small as compared with A, the distance D must be small compared with d. Numerous applications of this principle are made in hydraulic presses for compressing materials and forging steel, as also in jacks, accumulators, and hydraulic cranes.

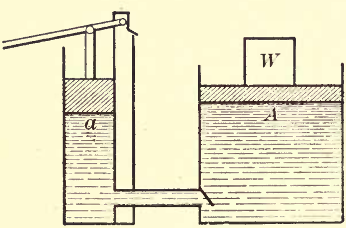


Fig. 3

In consequence of its fluidity the pressure existing at any point in a body of water is exerted in all directions with equal intensity. When water is confined by a bounding surface, as in a vessel, its pressure against that surface must be normal at every point, for if it were inclined, the water would move along the surface. When water has a free surface, the unit-pressure at any depth depends only on that depth and not on the shape of the vessel. Thus in the second diagram of Fig. 2 the unit-pressure at C produced by the smaller column of water aC is the same as that caused by the larger column AC, and the total vertical pressure on the upper side of the base B is the product of its area into the unit-pressure caused by the depth AB.

Prob. 5 *A*. What upward pressure is on the lower side of the base B in Fig. 2? Explain why this is different from the downward pressure on the upper side of the base B.

Prob. 5 *B*. The piston a in Fig. 3 is 2¼ inches in diameter while A is 17½ inches. When a moves 3½ inches, how much does A move?

ART. 6. HEAD AND PRESSURE

The free surface of water at rest is perpendicular to the direction of the force of gravity, and for bodies of water of small extent this surface may be regarded as a plane. Any depth below this plane is called a "head," or the head upon any point is its vertical depth below the level surface. In Art. 5 it was seen that the unit-pressure at any depth depends only on the head and not on the shape of the vessel. Let h be the head and w the weight of a cubic unit of water; then at the depth h one horizontal square unit bears a pressure equal to the weight of a column of water whose height is h, and whose cross-section is one square unit, or wh. But the pressure at this point is exerted in all directions with equal intensity. The unit-pressure p at the depth h then is wh, and the depth, or head, for a unit-pressure p is p/w, or

$$p = wh \qquad\qquad h = p/w$$

When h is expressed in feet and p in pounds per square foot, these formulas become, using the mean value of w,

$$p = 62.5\,h \qquad\qquad h = 0.016\,p$$

Thus pressure and head are mutually convertible, and in fact one is often used as synonymous with the other, although really each is proportional to the other. Any unit-pressure p can be regarded as produced by a head h, which is frequently called the "pressure-head."

In engineering work p is usually taken in pounds per square inch, while h is expressed in feet. Thus the pressure in pounds per square foot is $62.5\,h$, and the pressure in pounds per square inch is $1/144$ of this, or

$$p = 0.434\,h \qquad\qquad h = 2.30\,p \tag{6}$$

These rules may be stated in words as follows:

1 foot head corresponds to 0.434 pounds per square inch.
1 pound per square inch corresponds to 2.304 feet head.

These values, be it remembered, depend upon the assumption that 62.5 pounds is the weight of a cubic foot of water, and hence are liable to variations in the third significant figure (Art. 3).

Prob. 6 *A*. What is the pressure of the water in atmospheres at the bottom of an ocean 4 miles deep?

Prob. 6 *B*. How many pounds per square inch correspond to a head of 230 feet? How many feet head correspond to a pressure of 100 pounds per square inch?

ART. 7. LOSS OF WEIGHT IN WATER

It is a familiar fact that bodies submerged in water lose part of their weight; a man can carry under water a large stone which would be difficult to lift in air, and timber when submerged has a negative weight or tends to rise to the surface. The following is the law of loss which was discovered by Archimedes, about 250 B.C., when considering the problem of King Hiero's crown:

The weight of a body submerged in water is less than its weight in air by the weight of a volume of water which is equal to the volume of the body.

To demonstrate this, consider that the submerged body is acted upon by the water pressure in all directions, and that the horizontal components of these pressures must balance. Any vertical elementary prism is subjected to an upward pressure upon its base which is greater than the downward pressure upon its top. Let h_1 be the head on the top of the elementary prism in Fig. 4 and h_2 that on its base, and a the cross-section of the prism; then the downward pressure is wah_1 and the upward pressure is wah_2. The difference of these, $wa(h_2-h_1)$ is the resultant upward water pressure, and this is equal to the weight of a column of water whose cross-section is a and whose height is that of the elementary prism. Extending this theorem to all the elementary prisms, the theorem is demonstrated.

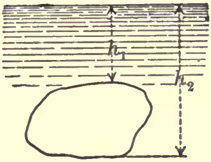


Fig. 4

It is important to regard this loss of weight in constructions under water. If, for example, a dam of loose stones allows the water to percolate through it, its weight per cubic foot is less than its weight in air, so that it can be more easily moved by horizontal forces. Since stone weighs about 150 pounds per cubic foot in air, its weight in water is only about $150-62=88$ pounds per cubic foot.

The ratio of the weight of a substance in air to that of an equal volume of water is called the specific gravity of the substance, and for a solid heavier than water this is easily computed from the law of Archimedes after weighing a piece of it in air and then in water. Thus, let W be the weight of a body in air and W_1 the weight in water, then $W - W_1$ is the weight of an equal volume of water, and the specific gravity of the body is $W/(W - W_1)$. Another method is to obtain the weight w_1 of a cubic unit of a substance and divide it by the weight w of a cubic unit of water.

Prob. 7 *A*. A piece of lead weighs 6.45 pounds in air and 5.88 pounds in water. Compute the specific gravity of lead.

Prob. 7 *B*. What is the specific gravity of steel when a cubic foot of it weighs 490 pounds?

ART. 8. PRESSURE ON SUBMERGED SURFACES

The total normal pressure on any immersed surface may be found by the following theorem:

> The total normal pressure is equal to the product of the weight of a cubic unit of water, the area of the surface, and the head on its center of gravity.

To prove this let A be the area of the surface in Fig. 5, and imagine it to be composed of elementary areas,

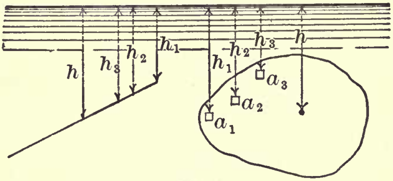


Fig. 5

a_1, a_2, a_3, etc., each of which is so small that the unit-pressure over it may be taken as uniform; let h_1, h_2, h_3, etc., be the heads on these elementary areas, and let w

denote the weight of a cubic unit of water. The unit-pressures at the depths h_1, h_2, h_3, etc., are wh_1, wh_2, wh_3, etc. (Art. 6), and hence the normal pressures on the elementary areas, a_1, a_2, a_3, etc., are wa_1h_1, wa_2h_2, wa_3h_3, etc. The total normal pressure P on the entire surface then is

$$P = w(a_1h_1 + a_2h_2 + a_3h_3 + \text{etc.})$$

Now let h be the head on the center of gravity of the surface; then, from the definition of the center of gravity,

$$a_1h_1 + a_2h_2 + a_3h_3 + \text{etc.} = Ah$$

Therefore the normal pressure is

$$P = wAh \tag{8}$$

which proves the theorem as stated.

This rule applies to all surfaces, whether plane, curved, or warped, and however they be situated with reference to the water surface. Thus the total normal pressure upon the surface of an immersed cylinder remains the same whatever be its position, provided the depth of the center of gravity of that surface be kept constant. It is best to take h in feet, A in square feet, and w as 62.5 pounds per cubic foot; then P will be in pounds. When surfaces are given whose centers of gravity are difficult to determine, they should be divided into simpler surfaces, and then the total normal pressure is the sum of the normal pressures on the separate surfaces.

The normal pressure on the base of a vessel filled with water is equal to the weight of a cylinder of water whose base is the base of the vessel, and whose height is the depth of water. Only in the case of a vertical cylinder does this become equal to the weight of the water, for the pressure on the base of a vessel depends upon the depth of water and not upon the shape of the vessel.

Also in the case of a dam, the depth of the water and not the size of the pond, determines the amount of pressure.

The pressure against an immersed plane surface in a given direction may be found by obtaining the normal pressure and computing its component in the required direction.

Prob. 8 *A*. A board 2 feet wide at the upper end, 4.5 feet wide at the lower end, and 6 feet long, is immersed vertically in water with the upper end in the water surface. Compute the normal pressure on each side of the board.

Prob. 8 *B*. A circular plate 5 feet in diameter is immersed so that the head on its center is 18 feet, its plane making an angle of 30° with the vertical. Compute the horizontal and vertical pressures upon one side of it.

ART. 9. CENTER OF PRESSURE ON RECTANGLES

The center of pressure on a surface immersed in water is the point of application of the resultant of all the normal pressures upon it. The simplest case is the following:

When a rectangle is placed with one end in the water surface, the center of pressure is distant from that end two-thirds of the length of the rectangle.

This theorem will be proved by the help of Fig. 6. The rectangle, which in practice might be a board, is placed with its breadth perpendicular to the plane of the drawing, so that *AB* represents its edge. It is required to find the center of pressure *C*. For any head *h* the unit-pressure is *wh* (Art. 8), and hence the unit-pressures on one side of *AB* may be

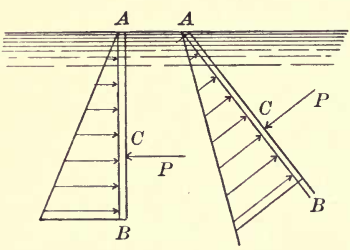


Fig. 6

graphically represented by arrows which form a triangle. Now when a force P equal to the total pressure is applied on the other side of the rectangle to balance these unit-pressures, it must be placed opposite to the center of gravity of the triangle. Therefore AC equals two-thirds of AB, and the rule is proved. The head on C is evidently also two-thirds of the head on B.

Another case is shown in Fig. 7, where the rectangle, whose length is B_1B_2, is wholly immersed, the head on B_1

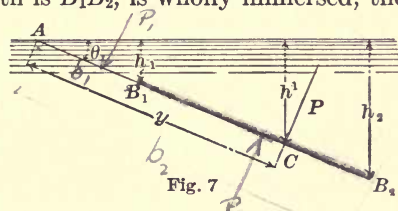


Fig. 7

being h_1, and on B_2 being h_2. Let $AB_1 = b_1$, $AC = y$, and $AB_2 = b_2$. Now the normal pressure P_1 on AB_1 is applied at the distance $\frac{2}{3}\,b_1$ from A, and the normal pressure P_2 on AB_2 is applied at the distance $\frac{2}{3}\,b_2$ from A. The normal pressure P on B_1B_2 is the difference of P_1 and P_2, or $P = P_2 - P_1$. Also, by taking moments about A as an axis,

$$P \times y = P_2 \times \tfrac{2}{3}\,b_2 - P_1 \times \tfrac{2}{3}\,b_1$$

Now, by Art. 7, the normal pressures P_2 and P_1 for a rectangle one unit in breadth are $P_2 = \frac{1}{2}wb_2h_2$ and $P_1 = \frac{1}{2}wb_1h_1$, whence the total normal pressure is $P = \frac{1}{2}w(b_2h_2 - b_1h_1)$. Inserting these in the above equation and replacing h_1 and h_2 by their values $b_1\sin\theta$ and $b_2\sin\theta$, the value of y is

$$y = \tfrac{2}{3}\cdot\ \frac{b_2^{\,3} - b_1^{\,3}}{b_2^{\,2} - b_1^{\,2}}$$

Or, if h' is the head on the center of pressure,

$$h' = \tfrac{2}{3}\cdot\ \frac{h_2^{\,3} - h_1^{\,3}}{h_2^{\,2} - h_1^{\,2}}$$

When $h_2=h_1=h$, this formula becomes indeterminate owing to the common factor h_2-h_1 in both numerator and denominator. Dividing out this common factor there is found $h'=h$.

Prob. 9 *A*. Find the center of pressure for Fig. 7 when $b_1=5$ and $b_2=16$ inches.

Prob. 9 *B*. A dam is 24 feet high and the water level behind it is 3 feet below the top. How far above the base is the center of pressure?

Prob. 9 *C*. A dam is 31 feet high and water runs over the top with a depth of 2 feet. How far above the base is the center of pressure?

ART. 10. PRESSURES ON GATES AND DAMS

In the case of an immersed plane the water presses equally upon both sides so that no disturbance of the equilibrium results from the pressure. But in case the water is at different levels on opposite sides of the surface the opposing pressures are unequal. For example, the cross-section of a self-acting tide-gate, built to drain a salt marsh, is shown in Fig. 8. On the ocean side there is a head of h_1 above the sill, which gives for every linear foot of the gate the horizontal pressure

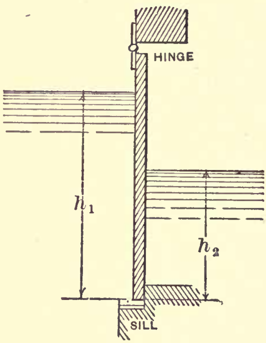


Fig. 8

$$P_1=w\times h_1\times \tfrac{1}{2}h_1=\tfrac{1}{2}wh_1^2$$

which is applied at the distance $\tfrac{1}{3}h_1$ above the sill. On the other side the head on the sill is h_2, which gives the horizontal pressure $P_2 = \frac{1}{2}wh_2^2$ acting in the opposite direction to that of P_1. The resultant horizontal pressure is

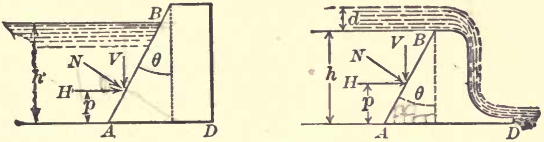
$$P = P_1 - P_2 = \tfrac{1}{2}w(h_1^2 - h_2^2)$$

and if z is the distance of the point of application of P above the sill, the equation of moments is

$$Pz = P_1 \times \tfrac{1}{3}h_1 - P_2 \times \tfrac{1}{3}h_2$$

from which z can be computed. For example, if h_1 is 7 feet and h_2 is 4 feet, the resultant pressure on one linear foot of the gate is found to be 1031 pounds and its point of application to be 2.82 feet above the sill. The action of this gate in resisting the water pressure is like that of a beam under its load, the two points of support being at the sill and the hinge.

When the water level behind a masonry dam is lower than its top, as in Fig. 9, the water pressure N on the



Figs. 9 and 10

back is normal to the plane AB and for computations this may be resolved into horizontal and vertical components H and V. Let h be the height of water above the base, θ the angle which the back makes with the vertical, then from Art. 8 the values of these pressures, for one linear unit of the dam, are

$$H = \tfrac{1}{2}wh^2 \qquad V = \tfrac{1}{2}wh^2 \tan\theta \qquad N = \tfrac{1}{2}wh^2 \sec\theta$$

and from Art. 9 the point of application of these pressures is at a distance $\frac{1}{3}h$ above the base. Except in the case of hollow dams only the horizontal component H need usually be considered, since the neglect of V is on the side of safety.

When the water runs over the top of a dam, as in Fig. 10, let h be the height of the dam and d the depth of water on its crest. Then

$$H = \tfrac{1}{2}wh(h+2d) \qquad V = H\tan\theta \qquad N = H\sec\theta$$

and the point of application above the base BD is

$$p = \frac{h+3d}{h+2d} \cdot \tfrac{1}{3}h$$

which always lies between $\frac{1}{3}h$ and $\frac{1}{2}h$.

Prob. 10 *A*. Find the greatest pressure on the hinge in Fig. 9 when the height of water is $6\tfrac{1}{2}$ feet and the length of the gate is 18 feet.

Prob. 10 *B*. For a dam like Fig. 11 let $h=64$ and $d=10$ feet, while the batter of the back is 4 on 1. Compute values of H, V, N for one linear foot of the dam.

Art. 11. Numerical Computations

The numerical work of computation should not be carried to a greater degree of refinement than the data of the problem warrant. For instance, in questions relating to pressures, the data are uncertain in the third significant figure, and hence more figures than three in the final result must be delusive. Thus, let it be required to compute the number of pounds of water in a box containing 307.37 cubic feet. Taking the mean value 62.5 pounds as the weight of one cubic foot, the multiplication gives the result 19 210.625 pounds, but evidently the decimals here have no precision, since the last figure in 62.5 is not accurate, and is likely to be less than 5, depending upon the impurity of the water and its temperature. The proper answer to this problem is 19 200 pounds, or perhaps 19 210 pounds, and this is to be regarded as a probable average result.

As this book is mainly intended for the use of students, a word of advice directed especially to them may not be inappropriate. It will be necessary for them, in order to gain a clear understanding of hydraulic science, or of any other engineering subject, to solve many numerical problems, and in this a neat and systematic method should be cultivated. The practice of performing computations on any loose scraps of paper that may happen to be at hand should be at once discontinued by every student who has followed it, and he should hereafter solve his problems in a special book provided for that purpose, and accompany them by such explanatory remarks as may seem necessary in order to render the solutions clear. Such a note-book, written in ink, and containing the fully worked out solutions of the examples and problems given in these pages, will prove of great value to every student who makes it. Before beginning the solution of a problem a diagram should be drawn whenever it is possible, for a diagram helps the student to clearly understand the problem, and a problem thoroughly understood is half solved. Before commencing the numerical work, it is also well to make a mental estimate of the final result.

The following problems will give a review of some of the principles and facts of this chapter:

Prob. 11 *A*. What is the weight in pounds of one Imperial gallon of water?

Prob. 11 *B*. How many pounds of water can be put into a vessel which contains 307.36 cubic feet?

Prob. 11 *C*. When one cubic foot of water falls each second from a vertical height of 33 feet, what horse-power can be delivered by a motor which utilizes 80 percent of the energy?

Prob. 11 *D*. A diver descends under water to a depth of 102 feet. What is the pressure upon him in atmosphere?

Prob. 11 *E*. What is weight in pounds of a pint of water? Does this agree with the old saying "a pint is a pound the world round"?

Prob. 11 *F*. What is the weight of a cubic foot of a substance whose specific gravity is 0.73?

Prob. 11 *G*. A rectangle is immersed in water with one end in the surface, its size being 2×12 feet, and its inclination to the vertical being 30°. Find the horizontal pressure upon it.

CHAPTER 2

THEORETIC HYDRAULICS

ART. 12. LAWS OF FALLING BODIES

Theoretic Hydraulics treats of the flow of water when unretarded by opposing forces of friction. The motion of water through orifices and pipes is produced by the force of gravity. This force is proportional to the acceleration of the velocity of a body falling freely in a vacuum; that is, to the increase in velocity in one second. Acceleration is measured in feet per second per second, so that its numerical value represents the number of feet per second which have been gained in one second. The letter g is used to denote the acceleration of a falling body near the surface of the earth. In pure mechanics g is found in all formulas relating to falling bodies; for instance, if a body falls from rest through the height h, it attains in a vacuum a velocity equal to $\sqrt{2gh}$. In hydraulics g is found in all formulas which express the laws of flow of water under the influence of gravity.

The quantity 32.16 feet per second per second is an approximate value of g which is used in this book. It is, however, well known that the force of gravity is not of constant intensity over the earth's surface, but is greater at the poles than at the equator, and also greater at the sea level than on high mountains. For sea level at the pole g has its greatest value, 32.26 feet per second per second. For a mountain 10 000 feet high at the equator g is 32.06 feet per second per second. These extreme values do not widely differ from the mean value 32.16 feet per second per second.

When a falling body has the initial velocity u at the beginning of the time t, its velocity at the end of this time is $V=u+gt$ and the distance passed over in that time is $h=ut+\frac{1}{2}gt^2$. Eliminating t from these equations gives

$$V=\sqrt{2gh+u^2} \qquad \text{or} \qquad h=(V^2-u^2)/2g \quad (12)$$

as the relations between V and h for this case. These formulas are also true whatever be the direction of the initial velocity u. When the initial velocity u is zero,

$$V=\sqrt{2gh} \qquad \text{or} \qquad h=V^2/2g$$

which are the formulas for a body falling from a position of rest with respect to the earth.

The general case of a body of weight W moving toward the earth is represented in Fig. 11. When the body is at A, it is at a height h_1 above a certain horizontal plane and has the velocity v_1, its potential energy being Wh_1 and its kinetic energy $Wv_1^2/2g$. When it has arrived at B its height above the plane is h_2 and its velocity is v_2, its potential energy being Wh_2 and its kinetic energy $Wv_2^2/2g$. In the first position the sum of its potential and kinetic energy with respect to the given horizontal plane is $W(h_1+v_1^2/2g)$, and in the second position the sum of these energies is $W(h_2+v_2^2/2g)$. If no energy has been lost between the two positions, these two expressions are equal, and hence

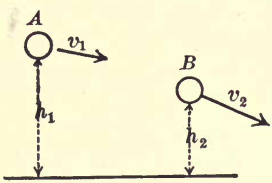


Fig. 11

$$h_1+\frac{v_1^2}{2g}=h_2+\frac{v_2^2}{2g}$$

This equation is the simplest form of Bernouilli's theorem (Art. 20). It contains two heights and two velocities, and when three of these quantities are given the fourth can be found.

Prob. 12 *A*. What is the velocity of *A* in Fig. 11, when it strikes the ground, if direction of v_1 is vertically upward?

Prob. 12 *B*. A body enters a room through the ceiling with a velocity of 17 feet per second, and in a direction making an angle of 30° with the vertical. If the height of the room is 16 feet, find the velocity of the body as it strikes the floor, resistances of the air being neglected.

ART. 13. VELOCITY OF FLOW FROM ORIFICES

When an orifice is opened, either in the base or side of a vessel containing water, the water flows out with a velocity which is greater for high heads than for low heads. The theoretic velocity of flow is given by the theorem established by Torricelli in 1644:

> The theoretic velocity of flow from the orifice is the same as that acquired by a body after having fallen from rest in a vacuum through a height equal to the head of water on the orifice.

One proof of this theorem is by experience. When a vessel is arranged, as in the first diagram of Fig. 12, so that a jet of water from an orifice is directed vertically upward, it is known that it never attains to the height of the level of the water in the vessel, although under favorable conditions it nearly reaches that level.

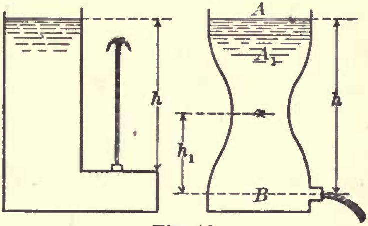


Fig. 12

It may hence be inferred that the jet would actually rise to that height were it not for the resistance of the air and the friction of the edges of the orifice. Now, since the velocity required to raise a body vertically to a certain height is the same as that acquired by it in falling from rest through that height, the theorem is established.

For any orifice, therefore, whether its plane is horizontal, vertical, or inclined, provided the head h is so large that it has practically the same value for all parts of the orifice, the relation between the velocity V of the issuing water and the head h is

$$V = \sqrt{2gh} \qquad \text{or} \qquad h = V^2/2g \tag{13}$$

the first of which gives the theoretic velocity of flow due to a given head, while the second gives the theoretic head that will produce a given velocity. The term "velocity-head" will generally be used to designate the expression $V^2/2g$, this being the height to which the jet would rise if it were directed vertically upward and there were no frictional resistances. Using for g the mean value 32.16 feet per second per second, these formulas become

$$V = 8.020\sqrt{h} \qquad \text{or} \qquad h = 0.01555V^2$$

in which h must be in feet and V in feet per second.

This discussion applies not only to water, but to any liquid. The direction of the velocity of exit is normal to the plane of the orifice.

√ Prob. 13 *A*. What velocity directed upward from a horizontal orifice will cause a jet to rise to the height of 16 feet?

√ Prob. 13 *B*. What is the theoretic velocity of a jet when under a head of one-tenth of a foot?

ART. 14. FLOW UNDER PRESSURE

When the level of the water in a vessel is subjected to a pressure p_o the velocity of flow from the orifice is increased It is also increased when the orifice is under a pressure p_1 which is less than that of the atmosphere. Let h be the head of actual water on the orifice, h_o the head of water which will produce the pressure p_o, and h_1 the head which will produce $p-p_1$, where p is the atmospheric pressure. Then the velocity of flow from the orifice is

$$V=\sqrt{2g(h+h_o+h_1)}$$

Usually p_o and p_1 are given in pounds per square inch, while h_o and h_1 are required in feet; then

$$h_o=2.304p_o \qquad\qquad h_1=2.304(14.7-p_1)$$

As an illustration let the cylindrical tank in Fig. 13 be 2 feet in diameter, and upon the surface of the water let there be a tightly fitting piston which with the load W weighs 3000 pounds. At the depth 8 feet below the water level are three small orifices: one at A, upon which there is an exterior head of water of 3 feet; one not shown in the figure, which discharges directly into the atmosphere; and one at C, where the discharge is into a vessel in which the air pressure is only 10 pounds per square inch. It is required to determine the velocity of efflux from each orifice. The head h_o corresponding to the pressure on the upper water surface is

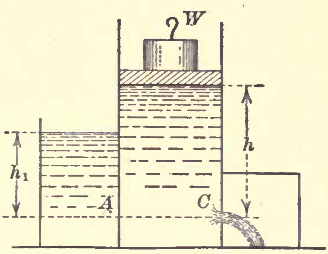


Fig. 13

$$h_o=\frac{p_0}{w}=\frac{3000}{3.142\times 62.5}=15.28 \text{ feet}$$

The head h_1 is -3 feet for the first orifice, 0 for the second, and 2.304 (14.7 − 10) = 10.83 feet for the third. The three theoretic velocities of outflow then are:

$$V = 8.02\sqrt{8+15.28-\ 3} \quad = 36.1 \text{ feet per second,}$$
$$V = 8.02\sqrt{8+15.28+\ 0} \quad = 38.7 \text{ feet per second,}$$
$$V = 8.02\sqrt{8+15.28+10.83} = 46.8 \text{ feet per second.}$$

In the case of discharge from an orifice under water, as at A in Fig. 13, the value of $h - h_1$ is the same wherever the orifice be placed below the lower level, and hence the velocity depends only upon the difference of level of the two water surfaces.

Prob. 14 *A*. Water under a head of 230 feet flows into a boiler whose gage reads 45 pounds per square inch. Find the theoretic velocity of the inflowing water.

Prob. 14 *B*. What is the theoretic velocity of flow from a small orifice in a boiler 1 foot below the water level when the steam-gage reads 60 pounds per square inch? What is the theoretic velocity when the gage reads 0?

ART. 15. INFLUENCE OF VELOCITY OF APPROACH

Thus far in the determination of the theoretic velocity from an orifice, the head upon it has been regarded as constant. But if the cross-section of the vessel is not large, the head can only be kept constant by an inflow of water, and this will modify the previous formulas. In this case the water approaches the orifice with an initial velocity. Let a be the area of the orifice and A the area of the horizontal cross-section of the vessel in Fig. 14. Let V be the velocity of flow through a and v be the vertical velocity of inflow through A. Let W be the weight

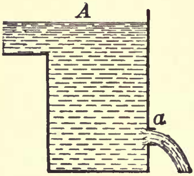


Fig. 14

of water flowing from the orifice in one second; then an equal weight must enter at A in one second in order to maintain a constant head h. The kinetic energy of the outflowing water is $W.V^2/2g$, and this is equal, if there be no loss of energy, to the potential energy $W.h$ of the inflowing water plus its kinetic energy $W.v^2/2g$, or,

$$W\frac{V^2}{2g} = Wh + W\frac{v^2}{2g}$$

Now since the same quantity of water Q passes through the two areas in one second, $Q = aV = Av$, whence $v = V.a/A$. Inserting this value of v in the equation of energy, there is found

$$V = \sqrt{\frac{2gh}{1-(a/A)^2}}$$

which is always greater than the value $\sqrt{2gh}$.

The influence of the velocity of approach on the velocity of flow at the orifice can now be ascertained by assigning values to the ratio a/A. Thus,

for $a = \frac{1}{2}A$	$V = 1.154\sqrt{2gh}$
for $a = \frac{1}{3}A$	$V = 1.061\sqrt{2gh}$
for $a = \frac{1}{5}A$	$V = 1.021\sqrt{2gh}$
for $a = \frac{1}{10}A$	$V = 1.005\sqrt{2gh}$

It is here seen that the common formula (13) is in error 2.1 percent when $a = \frac{1}{5}A$, if the head be maintained constant by a uniform vertical inflow at the water surface, and 0.5 percent when $a = \frac{1}{10}A$. Practically, if the area of the orifice is less than one-tenth of the cross-section of the vessel, the error in using the formula $V = \sqrt{2gh}$ is too small to be noticed, and fortunately most orifices are smaller in relative size than this.

Prob. 15 *A*. Show, when a is small compared with unity, that $1/(1+a) = 1-a$ and also that $1/\sqrt{1-a} = 1 + \frac{1}{2}a$.

$1 - (1+a)(1-a)$

Prob. 15 *B*. Show that a closely approximate expression for the theoretic velocity V, taking into account velocity of approach, is $V=(1+\frac{1}{2}(a/A)^2)\sqrt{2gh}$.

ART. 16. THE PATH OF A JET

When a jet of water issues from a small orifice in the vertical side of a vessel or reservoir, its direction at first is horizontal, but the force of gravity immediately causes the jet to move in a curve. Let x be the abscissa and y the ordinate of any point of the curve, measured from the orifice as an origin, as seen in Fig. 15. The effect of the impulse at the orifice is to cause the space x to be described uniformly in a certain time t, or, if v be the velocity of flow, $x=vt$. The effect of the force of gravity is to cause the space y to be described in accordance with the laws of falling bodies (Art. 12), or $y=\frac{1}{2}gt^2$. Eliminating t from these two equations, and replacing v^2 by its theoretic value $2gh$, gives

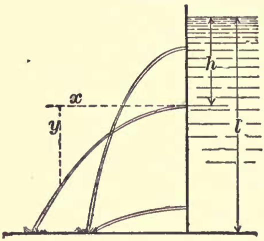


Fig. 15

$$y=gx^2/2v^2=x^2/4h$$

which is the equation of a parabola whose axis is vertical. The horizontal range of the jet for any given ordinate y is found from the equation $x^2=4hy$. When the height of the vessel is l, the horizontal range on the plane of the base hence is $x=2\sqrt{h(l-h)}$. The value of x is 0 when $h=0$ and also when $h=l$; it is a maximum when $h=\frac{1}{2}l$. Hence the greatest range is from an orifice at the mid-height of the vessel.

A more general case is that shown in Fig. 16 when the side AD of the vessel is inclined to the vertical at the angle θ. Here the jet issues normal to AD, rises to a highest point C, and then curves downward. If x and y are horizontal abscissa and vertical ordinate measured from A, it may be shown that the equation of the curve is

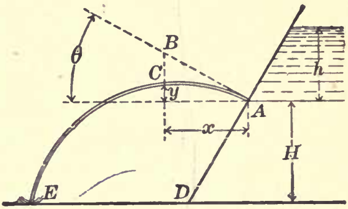


Fig. 16

$$y = x\tan\theta - x^2\sec^2\theta/4h$$

which is also the equation of a common parabola.

Prob. 16 *A*. For Fig. 15 let $h=6$ feet and $l=18$ feet. Compute the horizontal range of the jet on a horizontal plane one foot above the base.

Prob. 16 *B*. For Fig. 16 let $h=10$ feet and $H=6$ feet. Compute the velocity with which the jet strikes the horizontal plane *DE*.

ART. 17. ENERGY OF A JET

Let a jet or stream of water have the velocity v, and let W be the weight of water per second passing any given cross-section. The kinetic energy of this moving water is the same as that stored up by a body of weight W falling freely under the action of gravity through a height h and thereby acquiring the velocity v. Thus, if K represents kinetic energy per second,

$$K = Wh = W \colon v^2/2g$$

Now if a be the area of the cross-section and w the weight of a cubic unit of water, W is the weight of a prism of water of length v and cross-section a, or $W = wav$, whence

$$K = wav^3/2g \qquad (17)$$

and accordingly the energy which a jet can yield in one second is directly proportional to its cross-section and to the cube of its velocity. The term "power" is often used to express energy per second, and when K is in foot-pounds per second, the horse-power that a jet can yield is ascertained by dividing K by 550. Hence the horse-powers of jets of the same cross-section vary as the cubes of their velocities. For example, if the velocity of a jet be doubled, the cross-section remaining the same, the horse-power is made eight times as great. The term "energy of a jet" is often used in hydraulics for brevity, but it always means energy per second of the jet; that is, the power of the jet.

The expressions just deduced give the maximum work which can be obtained from the jet in one second, but this, in practice, can never be fully utilized. It is the constant aim of inventors so to arrange the conditions that the work realized may be as near the theoretic energy as possible. The "efficiency" of an apparatus for utilizing the power of water is the ratio of the work k actually utilized to the theoretic energy, or the efficiency e is

$$e = k/K$$

The greatest possible value of e is unity, but this can never be attained, owing to the imperfections of the apparatus and the frictional resistances. Values greater than 0.90 have, however, been obtained; that is, 90 percent or more of the theoretic power of the water has been utilized in some of the best forms of hydraulic motors.

For example, let water issue from a pipe 2 inches in diameter with a velocity of 40 feet per second. The cross-section in square feet is 3.142/144, and the kinetic energy of the jet in foot-pounds per second is

$$K = 0.01555 \times 62.5 \times 0.0218 \times 40^3 = 1357$$

which is 24.6 horse-powers. If this jet operates a motor yielding 17.7 horse-powers, the efficiency of the apparatus is $17.7/24.6=0.72$, or 72 percent of the theoretic energy is utilized.

Prob. 17 *A*. When water issues from a pipe with a velocity of 3 feet per second, its kinetic energy is sufficient to generate 1.3 horse-powers. What is the horse-power when the velocity becomes 6 feet per second?

Prob. 17 *B*. What horse-power can be furnished by a jet issuing from a nozzle 1¼ inches in diameter with a velocity of 80 feet per second, when the efficiency of the motor is 80 percent?

ART. 18. IMPULSE AND REACTION OF A JET

When a stream or jet is in motion, delivering W pounds of water per second with the uniform velocity v, that motion may be regarded as produced by a constant force F, which has acted upon W for one second and then ceased. In this second the velocity of W has increased from o to v, and the space $\frac{1}{2}v$ has been described. Consequently the work $F \times \frac{1}{2}v$ has been imparted to the water by the force F. But the kinetic energy of the moving water is $W.v^2/2g$, and hence by the law of conservation of energy $F \times \frac{1}{2}v = W \times v^2/2g$, from which the constant force is

$$F = W.v/g$$

This value of F is called the dynamic pressure or the impulse of the jet. As W is in pounds per second, v in feet per second, and g in feet per second per second, the value of F is in pounds.

The reaction of a jet upon a vessel occurs when water flows from an orifice. This reaction must be equal in value and opposite in direction to the impulse. In the direction of the jet the impulse produces motion, in the opposite direction it produces an equal pressure which tends to move the vessel backward. The force of reaction of a jet is hence equal to the impulse but is opposite in direction. For example (Fig. 17), let a vessel containing water be suspended at A so that it can swing freely, and let an orifice be opened in its side at B. The head of water at B causes a pressure which acts toward the left and causes W pounds of water to move during every second with the velocity of v feet per second, and which also acts toward the right and causes the vessel to swing out of the vertical; the first of these forces is the impulse, and the second is the reaction of the jet. If a force R be applied on the right of a vessel so as to prevent the swinging, its value is

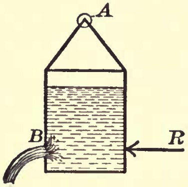


Fig. 17

$$R = F = W.v/g$$

and this is the formula for the reaction of the jet.

The impulse or reaction of a jet issuing from an orifice is double the hydrostatic pressure on the area of the orifice. Let h be the head of water, a the area of the orifice, and w the weight of a cubic unit of water; then, by Art. 8, the normal pressure when the orifice is closed is wah. When the orifice is opened, the weight of water issuing per second is $W = wav$, and hence the impulse or reaction of the jet is

$$R = F = wav.v/g = 2wa.v^2/2g = 2wah$$

which is double the hydrostatic pressure. This theoretic conclusion has been verified by many experiments.

When a jet impinges normally on a plane, it produces a dynamic pressure on that plane equal to the impulse

F, since the force required to stop W pounds of water in one second is the same as that required to put it in motion. Again, if a stream moving with the velocity v_1 is retarded so that its velocity becomes v_2, the impulse in the first instant is $W.v_1/g$, and in the second $W.v_2/g$. The difference of these, or

$$F_1 - F_2 = W(v_1 - v_2)/g$$

is a measure of the dynamic pressure which has been developed. It is by virtue of the pressure due to change of velocity that turbine wheels and other hydraulic motors transform the kinetic energy of moving water into useful work.

Prob. 19 *A*. If a stream of water issues horizontally from a vessel with a velocity of 15 feet per second, what reaction does it exert on the vessel?

Prob. 19 *B*. If a stream of water 3 inches in diameter issues from an orifice in a direction inclined downward 26° to the horizon with a velocity of 15 feet per second, find its horizontal reaction on the vessel.

ART. 19. THEORETIC DISCHARGE

The term "discharge" means the volume of water flowing in one second from a pipe or orifice, and the letter Q will designate the theoretic discharge; that is, the discharge as computed without considering the losses due to frictional resistances. When all the filaments of water issue from the pipe or orifice with the same velocity, the quantity of water issuing in one second is equal to the volume of a prism having a base equal to the cross-section of the stream and a length equal to the velocity. If this area is a and the theoretic velocity is V, then $Q = aV$ is the theoretic discharge. Taking a in square feet and V in feet per second, Q is in cubic feet per second.

For a small orifice on which the head h has the same value for all parts of the opening, the theoretic discharge is

$$Q = aV = a\sqrt{2gh} \tag{19}$$

and in English measures $Q = 8.02\, a\sqrt{h}$. For example, let a circular orifice 3 inches in diameter be under a head of 10.5 feet, and let it be required to compute Q. Here 3 inches $= 0.25$ feet, and the area of the circle is found to be 0.04909 square feet. From Art. 13 the theoretic velocity $V = 8.02 \times \sqrt{10.5} = 25.99$ feet per second. Accordingly the theoretic discharge is $0.04909 \times 25.99 = 1.28$ cubic feet per second.

The above formula for Q applies strictly only to horizontal orifices upon which the head h is constant, but it will be seen later that its error for vertical orifices is less than one-half of one percent when h is greater than double the depth of the orifice. Horizontal orifices are but little used, as it is more convenient in practice to arrange an opening in the side of a vessel than in its base. In applying the above formula to a vertical orifice, h is the vertical distance from its center to the free-water surface.

Since the theoretic velocity is always greater than the actual velocity, the theoretic discharge is a limit which can never be reached under actual conditions. Theoretically the discharge is independent of the shape of the orifice, so that a square orifice of area a gives the same theoretic discharge as a circular orifice of area a; it will be seen in Chap. 3 that this is not quite true for the actual discharge. It is supposed in the above formula that the velocity of a jet is the same in all parts of the cross-section, as this would be the case if h has the same value throughout the section were it not for the retarding influence of friction. If q is the actual discharge from any orifice and v the mean velocity in the area a, then

$\boxed{q=av}$, or the equation $\boxed{v=q/a}$ may be regarded as a definition of the term "mean velocity." The theoretic mean velocity is $\sqrt{2gh}$, but the actual mean velocity is smaller, as will be seen in Chap. 3.

Prob. 19 *A*. Compute the mean velocity in a pipe 12 inches in diameter when the discharge is 8.5 cubic feet per minute.

Prob. 19 *B*. Compute the theoretic head required to deliver 300 gallons of water per minute through an orifice 3 inches in diameter.

ART. 20. STEADY FLOW IN SMOOTH PIPES

When water flows through a pipe of varying cross-section and all sections are filled with water, the same quantity of water passes each section in one second. This is called the case of steady flow. Let q be this quantity of water and let v_1, v_2, v_3 be the mean velocities in three sections whose areas are a_1, a_2, a_3. Then

$$q=a_1v_1=a_2v_2=a_3v_3$$

This is called the condition for steady flow, and it shows that the velocities at different sections vary inversely as the areas of those sections. If v is the velocity at the end of the pipe where the area is a, then also $q=av$. When the discharge q and the areas of the sections have been measured, the mean velocities may be computed.

When a pipe is filled with water at rest, the pressure at any point depends only upon the head of water above that point. But when the water is in motion, it is a fact of observation that the pressure becomes less than that due to the head. The unit-pressure in any case may be measured by the height of a column of water. Thus if the water is at rest in the case shown in Fig. 18, and small tubes are inserted at the sections at a_1 and a_2, the water will rise in each tube to the same level as that of the water surface in the reservoir, and the pressures in the sections will be those due to the hydrostatic heads H_1 and H_2. But if the valve at the right be opened, the water levels in the small tubes will sink and the mean pressures in the two sections will be those due to the pressure-heads h_1 and h_2.

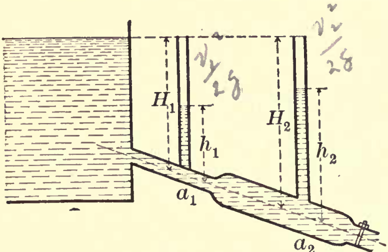


Fig. 18

Let W be the weight of water flowing in each second through each section of the pipe, and let v_1 and v_2 be the mean velocity in the section a_1 and a_2. When this water was at rest, the potential energy of pressure in the section a_1 was WH_1; when it is in motion, the energy in the section is the pressure energy Wh_1 plus the kinetic energy $W.v_1^2/2g$. If no losses of energy due to friction or impact have occurred, the energy in the two cases must be equal. The same reasoning applies to the section a_2, and hence

$$H_1 = h_1 + \frac{v_1^2}{2g} \qquad \text{and} \qquad H_2 = h_2 + \frac{v_2^2}{2g} \tag{20}$$

These equations exhibit the law of steady flow first deduced by Daniel Bernouilli in 1738, and hence often called Bernouilli's theorem; it may be stated in words as follows:

> At any section of a tube or pipe, under steady flow without friction, the pressure-head plus the velocity-head is equal to the hydrostatic head that obtains when there is no flow.

The pressure-head at any section hence decreases when the velocity of the water increases. To illustrate, let the depths of the centers of a_1 and a_2 be 6 and 8 feet below the water level, and let their areas be 1.2 and 2.4 square feet. Let the discharge of the pipe be 14.4 cubic feet per second. Then from (20) the mean velocity in a_1 is $v_1 = 14.4/1.2 = 12$ feet per second, which corresponds to a velocity head of $0.01555v^2 = 2.24$ feet, and consequently from the pressure-head in a_1 is $6.0 - 2.24 = 3.76$ feet. For the section a_2 the velocity is 6 feet per second and the velocity head is 0.56 feet, so that the pressure-head there is $8.0 - 0.56 = 7.44$ feet.

A negative pressure may occur when the velocity-head becomes greater than the hydrostatic head, for (20) shows that h_1 is negative when $v_1^2/2g$ exceeds H_1. A case of this kind is given in Fig. 19, where the section at A is so small that the velocity is greater than that due to the head H_1, so that if a tube be inserted at A, no water runs out; but if the tube be carried downward into a vessel of water, there will be lifted a column CD whose height is that of the negative pressure-head h_1. For example, let the cross-section of A be 0.4 square feet, and its head H_1 be 4.1 feet, while 8 cubic feet per second are discharged from the orifice below. Then the velocity at A is 20 feet per second, and the corresponding velocity-head is 6.22 feet. The pressure head at A then is, from the above theorem (20)

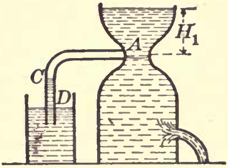


Fig. 19

$$h_1 = 4.1 - 6.22 = -2.12 \text{ feet}$$

and accordingly there exists at A an inward pressure

$$p_1 = -2.12 \times 0.434 = -0.92 \text{ pounds per square inch}$$

This negative pressure will sustain a column of water CD whose height is 2.12 feet. When the small vessel is placed so that its water level is less than 2.12 feet below A, water will be constantly drawn from the smaller to the larger vessel.

The siphon is a tube or pipe which rises higher than the water level in a vessel and discharges water at a lower level; Fig. 20 shows such a pipe. The velocity v in the pipe is uniform, being the same as that at the outlet end. For the highest point of the pipe, at a height H above the water level, the theoretic pressure-head is $h_1 = -H - v^2/2g$ which is always negative. Hence air tends to enter the upper part of the pipe under this negative pressure-head. For a point of the pipe, at a distance H_1 below the water level, the theoretic pressure-head is $h_1 = H_1 - v^2/2g$ which may be either positive or negative. Frictional resistances modify these algebraic expressions, as seen in Art. 38.

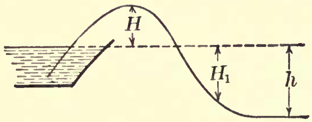


Fig. 20

Prob. 20 *A*. Compute the velocity-head of a jet 0.1 feet in diameter which discharges 2.5 cubic feet per minute.

Prob. 20 *B*. Compute the theoretic discharge from a vertical rectangular orifice 0.5 feet wide and 0.25 feet high, when the head on the top of the orifice is 0.375 feet.

Prob. 20 *C*. How many pounds of water flow in each second from a nozzle, 1 inch in diameter, when the velocity of the issuing water is 18 feet per second?

Prob. 20 *D*. Compute the theoretic discharge from an orifice one inch in diameter under a head of 18 inches.

Prob. 20 *E*. The hydrostatic pressure in a pipe is 80 pounds per square inch. What velocity must the water have to reduce this to 50 pounds per square inch?

CHAPTER 3

x to ad 28. P 52

FLOW FROM ORIFICES AND TUBES

ART. 21. THE STANDARD ORIFICE

Orifices for the measurement of water are usually placed in the vertical side of a vessel or reservoir, but may also be placed in the base. In the former case it is understood that the upper edge of the opening is completely covered with water; and generally the head of water on an orifice is at least three or four times its vertical height. The term "standard orifice" is here used to signify that the opening is so arranged that the water in flowing from it touches only a line, as would be the case in a plate of no thickness. In precise experiments the orifice may be in a metallic plate whose thickness is really small, as at A in Fig. 21, but more commonly it is cut in a board or plank, care being taken that the inner edge is a definite corner. It is usual to bevel the outer edges of the orifice, as at C, so that the escaping jet may by no possibility touch the edges except at the inner corner. This arrangement may be regarded as a standard apparatus for the measurement of water; for, as will be seen later, the discharge is modified when the inner corner is rounded, and different degrees of rounding give different discharges.

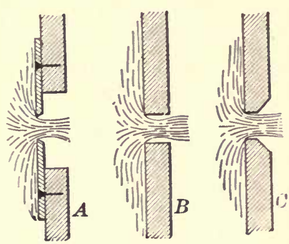


Fig. 21

The contraction of the jet which is always observed when water issues from a standard orifice, as described above, is a most interesting and important phenomenon. The appearance of such a jet under steady flow, issuing from a circular orifice, is that of a clear crystal bar whose beauty claims the admiration of every observer. The contraction of the jet is greatest at a distance from the plane of the orifice of about one-half its diameter. Beyond this section the jet enlarges in size if it is directed upward, but decreases in size if it is directed downward or horizontally.

The contraction of the jet is also observed in the case of rectangular and triangular orifices, its cross-section being similar to that of the orifice until the place of greatest contraction is passed. Fig. 22 shows in the top row cross-sections of a jet from a square orifice, in the middle row those from a triangular one, and in the third row those from an elliptical orifice. The left-hand diagram in each case is the cross-section of the jet near the place of greatest contraction, while the following ones are cross-sections at greater distances from the orifice, and the jets are supposed to be moving horizontally or nearly so.

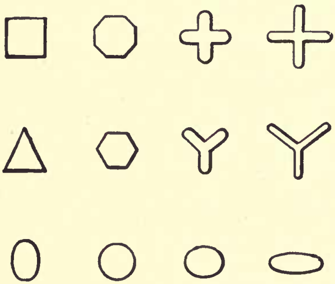


Fig. 22

Owing to this contraction, the discharge from a standard orifice is always less than the theoretic discharge, which, from Arts. 13 and 19, is expressed by $Q = a\sqrt{2gh}$ where a is the area of the orifice and h the head above its center. It is customary, then, to multiply the theoretic discharge by a number c, called the coefficient of discharge. Then the quantity of water flowing per second from the orifice, is cQ, or

$$q = ca\sqrt{2gh}$$

is the formula for the actual discharge from an orifice. A mean value of the coefficient c is 0.61, that is the actual discharge is 61 percent of the theoretic discharge.

Prob. 21 *A*. Compute the actual discharge from an orifice one inch in diameter under a head of 1.5 feet.

Prob. 21 *B*. The discharge from a certain orifice under a head of 3.5 feet is 1.357 cubic feet per second. What is the discharge when the head is 7.0 feet? Omit.

ART. 22. COEFFICIENT OF CONTRACTION

The coefficient of contraction is the number by which the area of the orifice is to be multiplied in order to give the area of the section of the jet at the place of its greatest contraction. Thus, if c' be the coefficient of contraction, a the area of the orifice, and a' the area of the contracted section of the jet, then

$$a' = c'a \tag{22}$$

The coefficient of contraction for a standard orifice is evidently always less than unity.

The only direct method of finding the value of c' is to measure by calipers the dimensions of the least cross-section of the jet. The size of the orifice can usually be determined with precision, as also the diameter of a circular jet. Let d and d' be the diameters of the areas a and a^1; then

$$c' = a'/a = (d'/d)^2$$

Therefore the coefficient of contraction is the square of the ratio of the diameter of the jet to that of the orifice. The first measurements were made in 1685 by Newton, who found the ratio of d' to d to be 21/25, which gives for c the value 0.71. Other experimenters have found values ranging from 0.57 to 0.66, but the best work indicates a range from 0.60 to 0.63.

The following mean value will be used in this book for orifices, and it should be kept in mind by the student:

$$\text{Coefficient of contraction } c' = 0.62$$

or, in other words, the minimum cross-section of the jet is 62 percent of that of the orifice. This value, however, undoubtedly varies for different forms of orifices and for the same orifice under different heads, but little is known regarding the extent of these variations or the laws that govern them. Probably c' is slightly smaller for circles than for squares, and smaller for squares than for rectangles, particularly if the height of the rectangle is long compared with its width. Probably also c' is larger for low heads than for high heads.

Prob. 22 *A*. The diameter of a circular orifice is 1.995 inches. Three measurements of the diameter of the contracted section of the jet gave 1.55, 1.56, and 1.59 inches. Find the mean coefficient of contraction.

Prob. 22 *B*. A circular orifice is 3 inches in diameter. What is the diameter of the jet at the contracted section?

ART. 23. COEFFICIENT OF VELOCITY

The coefficient of velocity is the number by which the theoretic velocity of flow from the orifice is to be multiplied in order to give the actual velocity at the least cross-section of the jet. Thus, if c_1 be the coefficient of velocity, V the theoretic velocity due to the head on the center of the orifice, and v the actual velocity at the contracted section, then

$$v = c_1 V = c_1 \sqrt{2gh} \tag{23}$$

The coefficient of velocity must be less than unity, since the force of gravity cannot generate a greater velocity than that due to the head.

The velocity of flow at the contracted section of the jet cannot be directly measured. To obtain the value of the coefficient of velocity, indirect observations have been taken on the path of the jet. Referring to Art. 16, it will be seen that when a jet flows from an orifice in the vertical side of a vessel, it takes a path whose equation is $y=gx^2/2v^2$, in which x and y are the coordinates of any point of the path measured from vertical and horizontal axes, and v is the velocity at the origin. Now placing for v its value $c_1\sqrt{2gh}$, and solving for c_1, gives

$$c_1=x/2\sqrt{hy}$$

Therefore c_1 becomes known by the measurement of the head h and the coordinates x and y. In making this experiment it would be well to have a ring, a little larger than the jet, supported by a stiff frame which can be moved until the jet passes through the ring. The flow of water can then be stopped, and the coordinates of the center of the ring determined. By placing the ring at different points of the path different sets of coordinates can be obtained. The value of x should be measured from the contracted section rather than from the orifice, since v is the velocity at the former point.

By this method various values of c_1 ranging from 0.97 to 0.99 have been found. As a mean value for standard orifices the following may be kept in the memory:

$$\text{Coefficient of velocity } c_1=0.98$$

or, the actual valocity of flow at the contracted section is 98 percent of the theoretic velocity. The value of c_1 is greater for high than for low heads.

Prob. 23. A jet issuing from the vertical side of a vessel has a range of 12.5 feet on a horizontal plane 2.82 feet below the orifice. The head being 14.38 feet, compute the coefficient of velocity.

ART. 24. COEFFICIENT OF DISCHARGE

The coefficient of discharge is the number by which the theoretic discharge is to be multiplied in order to obtain the actual discharge. Thus, if c is the coefficient of discharge, Q the theoretical and q the actual discharge per second, then

$$q = cQ \tag{24}$$

Here also the coefficient c is a number less than unity.

The coefficient of discharge can be accurately found by allowing the flow from an orifice to fall into a vessel of constant cross-section and measuring the heights of water by the hook gage (Art. 53). Thus q is known, and Q having been computed, then

$$c = q/Q$$

For example, a circular orifice of 0.1 foot diameter was kept under a constant head of 4.677 feet; during 5 minutes and 32⅕ seconds the jet flowed into a measuring vessel which was found to contain 27.28 cubic feet. Here the actual discharge was

$$q = 27.28/332.2 = 0.08212 \text{ cubic feet per second}$$

The theoretic discharge then is

$$Q = \pi \times 0.05^2 \times 8.02\sqrt{4.677} = 0.1361 \text{ cubic feet per second}$$

Then the coefficient of discharge is found to be

$$c = 0.08212/0.1361 = 0.604$$

In this manner thousands of experiments have been made upon different forms of orifices under different heads, for accurate knowledge regarding this coefficient is of great importance in practical hydraulic work.

In general c is greater for low heads than for high heads, greater for rectangles than for squares, and greater for squares than for circles. Its values range from 0.59 to 0.63 or higher, and as a mean to be kept in mind,

$$\text{Coefficient of discharge } c = 0.61$$

or, the actual discharge from a standard orifice is, on the average, about 61 percent of the theoretic discharge.

The coefficient c may be expressed in terms of the coefficients c' and c_1. Let a and a' be the areas of the orifice and the cross-section of the contracted jet, and Q and q the theoretic and actual discharge per second. Then, since $a'/a = c'$,

$$c = \frac{q}{Q} = \frac{a'c_1\sqrt{2gh}}{a\sqrt{2gh}} = \frac{a'c_1}{a} = c'c_1$$

and therefore the coefficient of discharge is the product of the coefficients of contraction and velocity.

These three coefficients are of much importance, not only for orifices but for tubes and weirs, although the numerical values may be different from those above stated. The coefficient of discharge is of greater value than the coefficients of contraction and velocity, since it is the quantity generally used in making measurements of water.

Prob. 24 *A*. What is the discharge in gallons per minute from a circular orifice one inch in diameter under a head of one foot, the coefficient of discharge being 0.609?

Prob. 24 *B*. The diameter of a contracted circular jet was found to be 0.79 inches, the diameter of the orifice being 1 inch. Under a head of 16 feet the actual discharge per minute was found to be 6.42 cubic feet. Find the coefficient of velocity.

ART. 25. CIRCULAR VERTICAL ORIFICES

Let a circular orifice of diameter d and area a be in the side of a vessel and let h be the head of water on its center. Then, from Art. 13, the theoretic mean velocity is $\sqrt{2gh}$, and from Art. 19 the theoretic discharge is

$$Q = a\sqrt{2gh} = \tfrac{1}{4}\pi d^2 \sqrt{2gh}$$

This formula, however, is not strictly exact, because the velocity in the lower part of the orifice is greater than that at the top, owing to the greater head. By the methods of higher mathematics the following more exact formula is derived

$$Q = [1 - 0.007812(d/h)^2]\tfrac{1}{4}\pi d^2 \sqrt{2gh}$$

which should be used when h is less than $2d$. For $h = d$ the quantity in the parenthesis is 0.992 and for $h = 2d$ it is 0.998. Hence the error in using the first formula is less than three-tenths of one percent when the head on the center of the orifice is greater than twice its diameter.

For most cases, then, the actual discharge from a circular vertical orifice of area a may be computed from

$$q = c \times a\sqrt{2gh} = c \times 8.02\ a\sqrt{h}$$

in which c is the coefficient of discharge. When h is smaller than two or three times the diameter of the orifice, and when precision is required, then

$$q = c[1 - 0.007812(d/h)^2]8.02\ a\sqrt{h}$$

Here h is the head in feet, and a is the area of the circular orifice in square feet; then q will be in cubic feet per second.

The following values of c for standard circular orifices are taken from a more extended table in Merriman's Treatise on Hydraulics (New York, 1912):

Head in feet,	$h=0.6$	0.8	1.0	2.0	6.0	20.0
For $d=0.02$ feet,	$c=0.655$	0.648	0.644	0.632	0.618	0.601
For $d=0.04$ feet,	$c=$ .630	.626	.623	.614	.607	.599
For $d=0.07$ feet,	$c=$ .618	.615	.612	.607	.602	.597
For $d=0.1$ feet,	$c=$ .613	.610	.608	.604	.600	.596
For $d=0.2$ feet,	$c=$ .601	.601	.600	.599	.598	.596
For $d=0.6$ feet,	$c=$ .593	.594	.595	.597	.597	.596
For $d=1.0$ foot	$c=$ .590	.591	.593	.595	.596	.594

As an example of the computation of discharge, let it be required to find q for a standard orifice 2 inches in diameter under a head of 4 feet. Here 2 inches $=0.167$ feet, so that c lies between the third and fourth lines from the foot of the table and also between the fourth and fifth columns. Interpolating first between the columns c lies between 0.602 and 0.5985; then interpolating between these two values it is found that the value of c required is 0.600. Then $a=0.02181$ square feet and the formula gives $q=0.210$ cubic feet per second, a value which is uncertain in the third decimal place, since the values of c have probably a like uncertainty.

Prob. 25 *A*. Find from the table the coefficient of discharge from a standard circular orifice under a head of 1.75 feet.

Prob. 25 *B*. Compute the coefficient of discharge for a case where 1.61 cubic feet of water flowed in 10 seconds from an orifice 2 inches in diameter under a head of 2.35 feet.

ART. 26. RECTANGULAR VERTICAL ORIFICES

When the size of an orifice in the side of a vessel is small compared with the head, the theoretic velocity of the outflowing water may be taken as $\sqrt{2gh}$, where h is the head on the center of the orifice. For a rectangular orifice under this condition the theoretic discharge is

$$Q = bd\sqrt{2gh}$$

where b is the width and d the depth of the orifice. When b is equal to d, the rectangle becomes a square.

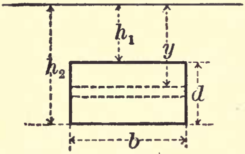


Fig. 23

For a more exact formula, let h_1 be the head on the upper edge of the orifice, h_2 that on the lower edge (Fig. 23). By the methods of higher mathematics it may be then found that

$$Q = \tfrac{2}{3} b\sqrt{2g}(h_2^{3/2} - h_1^{3/2})$$

which is the theoretic discharge through the rectangle. Placing in this $h_2 = h + \frac{1}{2}d$ and $h_1 = h - \frac{1}{2}d$ and developing by the binomial formula, it becomes

$$Q = \left[1 - \frac{(d/h)^2}{96} - \frac{(d/h)^4}{2048}\right] bd\sqrt{2gh}$$

and this shows that the discharge computed by using the first formula is always too great. For $h = d$, the quantity in the parenthesis is 0.989, and for $h = 2d$, it is 0.997. Accordingly, the error of the approximate formula is only three-tenths of one percent when the head on the center of the rectangle is twice the depth of the orifice.

For most cases, then, the actual discharge from a square vertical orifice may be approximately found from

$$q = c \times d^2\sqrt{2gh} = c \times 8.02 d^2\sqrt{h}$$

where d is the side of the square and c is the coefficient of discharge. When h is smaller than two or three times the side of the orifice, and when precision is required, then

$$q=c\left[1-\frac{(d/h)^2}{96}-\frac{(d/h)^4}{2048}-\right]d^2\sqrt{2gh}$$

In both cases the linear quantities are to be taken in feet, and then q will be in cubic feet per second.

Values of c for square vertical orifices as determined by numerous experiments are given in the following table:

Head in feet,	$h=0.6$	0.8	1.0	2.0	6.0	20.0
For $d=0.02$ feet,	$c=0.660$	0.652	0.648	0.637	0.623	0.606
For $d=0.04$ feet,	$c=$ .636	.631	.628	.619	.612	.604
For $d=0.07$ feet,	$c=$ .623	.620	.618	.612	.607	.602
For $d=0.1$ feet,	$c=$ .617	.615	.613	.608	.605	.602
For $d=0.2$ feet,	$c=$ .605	.605	.605	.605	.604	.602
For $d=0.6$ feet,	$c=$ .598	.600	.601	.604	.603	.601
For $d=1.0$ foot,	$c=$ .596	.597	.599	.602	.602	.600

Comparing this table with that in the last article it is seen that c for a square is always slightly larger than that for a circle having a diameter equal to the side of a square.

For a rectangle having its lowest side horizontal, the coefficient for approximate work may be taken the same as for a square orifice with side equal to the depth of the rectangular one; then the discharge is found by the above formulas for Q, these being multiplied by the proper coefficient c. The discharge thus computed is slightly too small.

Velocity of approach must be taken into account in computing the discharge in cases where great precision is required. See Art. 15.

Prob. 26 *A*. Compute the discharge for a standard orifice 2 inches square under a head of 4 feet.

Prob. 26 *B*. Compute the approximate discharge for a rectangular orifice 12 inches wide and 4 inches deep when the head on the top of the orifice is 32 inches.

ART. 27. SPECIAL ORIFICES

A submerged orifice is one which discharges under water under a head h as shown in Fig. 24. The theoretic discharge is the same for the same head h_1 whatever be the depth of the orifice below the lower water level. The theoretic velocity in all parts of the orifice is the same, as may be proved from Fig. 24, where the triangles ACD and BCE represent the distribution of pressure on AC and BC when the orifice is closed (Art. 8). Making CF equal to CE and drawing BF, the unit-pressure on BC is seen to have the constant value DF. Now when the orifice is opened, the velocity at any point depends on the unit-pressure there acting, and accordingly the theoretic velocity is uniform over the section. The coefficient of discharge c, when the inner edge is arranged like a standard orifice, ranges from 0.60 to 0.62. Submerged orifices are used for canal-locks, tide-gates, filter-beds, for the discharge of waste water through dams, and for the admission of water from a canal to a power-plant. The inner edges of such orifices are usually rounded, and the coefficient of discharge may then be higher than 0.9.

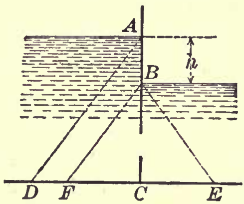


Fig. 24

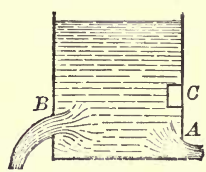


Fig. 25

When a vertical orifice has its lower edge at the bottom of the reservoir, as shown at A in Fig. 25, the particles of water flowing through its lower portion move in lines nearly perpendicular to the plane of the orifice, or the contraction of the jet does not form on the lower side. This is called a case of suppressed or incomplete contraction. The same thing occurs, but in a lesser degree, when the lower edge of the orifice is near the bottom, as shown at B. In like manner, when an orifice is placed so that one of its vertical edges is at or near a side of the reservoir, as at C, the contraction of the jet is suppressed upon one side, and when it is placed at the lower corner of the reservoir suppression occurs both upon one side and the lower part of the jet. Experiments show that for square orifices with contraction suppressed on one side the coefficient of discharge is increased about 3.5 percent, and with contraction suppressed on two sides about 7.5 percent. For a rectangular orifice with the contraction suppressed on the bottom edge the percentages are larger, being about 6 or 7 percent when the length of the rectangle is four times its height.

When the inner edge of the orifice is rounded, as shown in Fig. 26, the contraction of the jet is modified, and the discharge is increased. With a slight degree of rounding, as at A, a partial contraction occurs; but with a more complete rounding, as at C, the particles of water issue perpendicular to the plane of the orifice and there is no contraction of the jet. For a standard orifice with sharp inner edges (Art. 22) the mean value of c' is 0.62, but for an orifice with rounded edges c' may have any value between 0.62 and 1.0, depending upon the degree of round-

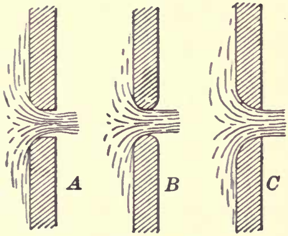


Fig. 26

ing. A rounded interior edge in an orifice is therefore always a source of error when the object of the orifice is the measurement of the discharge.

Prob. 27 *A*. When an orifice with rounded edges has a coefficient of velocity of 0.88 and a coefficient of discharge of 0.75, find the coefficient of contraction of the jet.

Prob. 27 *B*. Compute the probable discharge from a vertical orifice one foot square when the head on its upper edge is 4 feet, the contraction being suppressed on the lower edge. Compute the discharge for the same data when contraction is suppressed on all sides.

ART. 28. THE STANDARD SHORT TUBE

The standard short tube is a cylinder whose length is about three times its diameter through which water flows from a vessel. The inner edge of the tube has a sharp definite corner like that of the standard orifice. When a tube is about one or two diameters in length the stream passes through without touching the tube and the discharge is the same as that from the standard orifice. When it is lengthened sufficiently the stream fills the tube as in Fig. 27 and the discharge is much increased. By observations on glass tubes it is seen that the stream usually contracts at first and then expands. This contraction may be apparently destroyed by agitating the water and the entire tube is then filled, yet if a hole is bored in the tube near the inner end, water does not flow out but air enters, showing that a negative pressure exists there.

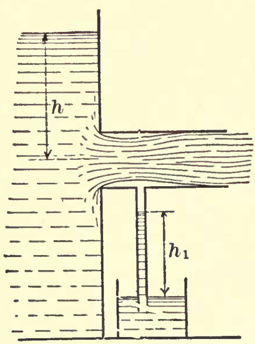


Fig. 27

Experiments show that the discharge of the standard short tube is usually from 30 to 36 percent greater than that of the standard orifice. Referring to the outer end of the tube where the section is completely filled, the mean coefficients are

Coefficient of contraction = 1.00
Coefficient of velocity = 0.82
Coefficient of discharge = 0.82

The cause of the increased discharge of the tube over the orifice is a partial vacuum, which causes a portion of the atmospheric head of 34 feet to be added to the head h, so that the flow at the contracted section occurs as if under the head $h+h_1$ (Fig. 27). The occurrence of this partial vacuum is attributed to the friction of the water on the air. When the flow begins, the stream is surrounded by air of the normal atmospheric pressure which is imprisoned as the stream fills the tube. The friction of the moving water carries some of this air out with it, thus rarefying the remaining air. This rarefaction, or negative pressure, is followed by an increased velocity of flow, and the process continues until the air around the contracted section is so rarefied that no more is removed, and the flow then remains permanent. The partial vacuum causes neither a gain nor loss of head, for although it increases the velocity-head at the contracted section, this must be later decreased in order to overcome the atmospheric pressure at the outer end of the tube. The experiments of Buff have proved that in an almost complete vacuum the discharge of the tube is but little greater than that of the orifice.

It will be shown that the negative pressure-head h_1 is about $\frac{3}{4}h$, as was first discovered experimentally by Venturi. The coefficient of contraction for the contracted section being 0.62 (Art. 22) and that at the outer end of the tube being 0.82, the velocity in the contracted section is

$$v=\frac{0.82}{0.62}\sqrt{2gh}=1.32\sqrt{2gh}$$

and hence the velocity-head in that section is

$$v^2/2g=1.75h$$

Accordingly from the theorem that pressure-head plus velocity-head equals total head (Art. 20) the pressure-head in the contracted section is $h-1.75h=-0.75$ which is the value of h_1 in Fig. 27. This conclusion does not hold for very high heads, since in no event can atmospheric pressure raise a column of water higher than about 34 feet (Art. 4).

Prob. 28 *A*. Show that the mean velocity-head of a stream from a standard orifice is 96 percent of the head.

Prob. 28 *B*. Show that the mean velocity-head of a stream from a standard short tube is 67 percent of the head.

ART. 29. VARIOUS FORMS OF TUBES

Conical converging tubes are used when it is desired to obtain a high efficiency in the energy of the stream of water. At *A*, Fig. 28, is shown a simple converging tube, consisting of a frustrum of a cone, and at *B* is a similar frustrum provided with a cylindrical tip.

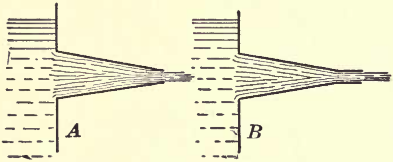


Fig. 28

The proportions of these converging tubes, or mouthpieces, vary somewhat in practice, but the cylindrical tip, when employed, is of a length equal to about 2½ times its inner diameter, while the conical part is eight or ten times the length of that diameter. The stream from a conical converging tube like A suffers a contraction at some distance beyond the end. The coefficient of discharge is higher than that of the standard tube, being generally between 0.85 and 0.95.

Inward projecting tubes, as a rule, give a less discharge than those whose ends are flush with the side of the

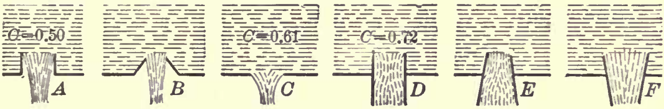


Fig. 29

reservoir, due to the greater convergence of the lines of direction of the filaments of water. At A and B, Fig. 29, are shown inward projecting tubes so short that the water merely touches their inner edges, and hence they may more properly be called orifices. Experiment shows that the case at A, where the sides of the tube are normal to the side of the reservoir, gives the minimum coefficient of discharge $c = 0.5$, while for B the value lies between 0.5 and that for the standard orifice at C. The inward projecting cylindrical tube at D has been found to give a discharge of about 72 percent of the theoretic discharge, while the standard tube gives 82 percent.

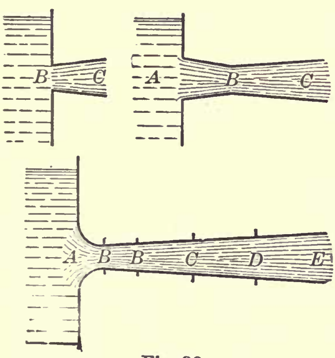


Fig. 30

In Fig. 30 is shown a diverging conical tube, BC, and two compound tubes. The compound tube ABC consists of two cones, the converging one AB being much shorter than the diverging one BC, so that the shape roughly approximates to the form of the contracted jet which issues from an orifice in a thin plate. In the tube AE the curved converging part AB closely imitates the contracted jet, and BB is a short cylinder in which all the filaments of the stream are supposed to move in lines parallel to the axis of the tube, the remaining part being a frustrum of a cone. The converging part of a compound tube is often called a mouthpiece and the diverging part an adjutage. Experiments were made by J. B. Francis on a tube BE of 4.1 feet long, the diameters at B and E being 0.102 and 0.321 feet, showed the coefficient of discharge for the section B to be 2.43 and that for the section E to be 0.24, the head being 1.36 feet. Here the velocity-head for the section B was $5.9h$ so that a negative pressure-head of $4.9h$ existed there. The explanation of this is similar to that given for the standard short tube in Art. 28.

Prob. 29 *A*. Find the coefficient of discharge for a tube whose diameter is one inch when the flow under a head of 9 feet is 22.1 cubic feet in 3 minutes and 30 seconds.

Prob. 29 *B*. When the coefficient of discharge of a tube is 0.98 and the coefficient of velocity of the jet is 0.995, compute the coefficient of contraction of the jet.

ART. 30. LOSSES OF HEAD

A jet of water flowing from an orifice with the velocity v has the velocity-head $v^2/2g$ which is always less than the pressure-head h. The difference of these is the head lost in friction and impact, or

$$h' = h - \frac{v^2}{2g}$$

which applies not only to an orifice but to any tube or pipe. For the orifice or tube $v = c_1\sqrt{2gh}$ where c_1 is the coefficient of velocity and hence

$$h' = (1 - c_1^2)h$$

which gives the lost head in terms of the total head, or

$$h' = \left(\frac{1}{c_1^2} - 1\right)\frac{v^2}{2g}$$

which gives the lost head in terms of the velocity-head of the jet.

In hydraulics the terms "energy" and "head" are often used as synonymous, although really energy is proportional to head. Thus the pressure-head that causes the flow is h and the velocity-head of the issuing jet is $v^2/2g$, and these are proportional to the theoretic and effective energies. For the standard orifice the mean value of c_1 is 0.98, and hence a mean value of c_1^2 is 0.96. The actual energy of a jet from such an orifice is hence about 96 percent of the theoretic energy, and the loss of energy or head is about 4 percent. This loss is due to the frictional resistance of the edges of the orifice, whereby the energy of pressure or velocity is changed into heat.

For the standard short tube the mean value of c_1 is 0.82 and hence a mean value of c_1^2 is 0.67. The actual energy of a jet from such a tube is hence about 0.67 percent of the theoretic energy, and the loss of energy or head is about 33 percent. This loss is due both to friction and to the impact of the expanding stream.

The above discussion shows that if jets are directed vertically upward from a standard orifice and tube, as in Fig. 31, the jet from the former rises to the height $0.96h$, while that from the latter rises to the height $0.67h$, where h is the head measured downward from the surface of water in the reservoir to the point of exit from the orifice. Hence, if the issuing streams are of the same diameter, the power derivable from the former is much greater than from the latter.

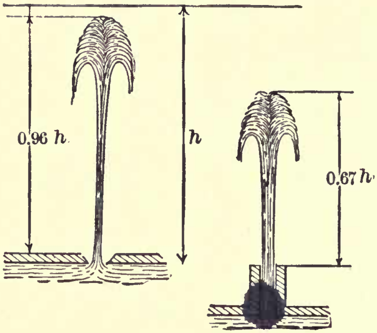


Fig. 31

Energy within a pipe or tube filled with moving water usually exists in two forms, in potential energy of pressure and in kinetic energy of motion, the former corresponding to pressure-head and the latter to velocity-head. Thus for the case of gradual contraction shown in Fig. 32, let two small vertical tubes, called piezometers, be inserted; the water rises in these to the heights h_1 and h_2, which are the pressure-heads at the two sections. The mean velocities at these sections being v_1 and v_2 the velocity-heads are $v_1^2/2g$ and $v_2^2/2g$. The weight of water flowing per second being W, the total energy in the first section is $W(h_1+v_1^2/2g)$ and that in the second section is $W\ (h_2+v_2^2/2g)$. The difference of these is the energy Wh' which is lost in friction and impact in passing from the first to the second section. Hence

$$h' = h_1 - h_2 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} \quad (30)$$

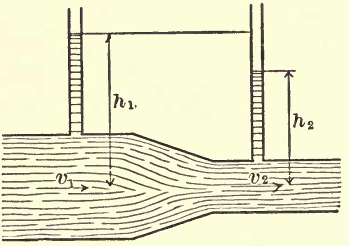


Fig. 32

which shows that for a horizontal tube or pipe the lost head h' between two sections is equal to the difference of the pressure-heads plus the difference of the velocity-heads. When the two sections are of the same size the velocities v_1 and v_2 are equal, since the same quantity of water passes each section in one second, and then $h' = h_1 - h_2$ or the lost head is equal to the difference of the pressure-heads.

For the case of gradual expansion of section shown in Fig. 33 the same reasoning applies and the same formula results. Here, however, h_1 is smaller than h_2, so that the difference $h_1 - h_2$ is negative, its numerical value being less than the difference between the velocity-heads. The above formula applies to all cases of loss of head between two sections of a horizontal tube or pipe. For the case when the pipe is inclined see Art. 37.

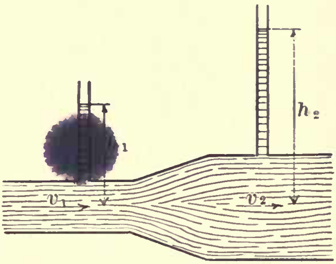


Fig. 33

Prob. 30 *A*. For an inward projecting tube (*D* in Fig. 31) the coefficient of discharge is 0.72. Show that the lost head is 0.52 h.

Prob. 30 *B*. In Fig. 32 let the areas of the two sections be 1.0 and 0.5 square feet, $h_1 = 3.250$ feet, $h_2 = 3.947$ feet, and $v_1 = 3.5$ feet per second. Compute the lost head h'.

ART. 31. SUDDEN CHANGES IN SECTION

When there is a sudden enlargement of section in a tube or pipe, as in Fig. 34, energy is lost in impact. In the section AB the pressure-head is h_1 and the velocity-head is $v_1^2/2g$, while in the section CD the pressure-head has the larger value h_2 and the velocity-head has the smaller value $v_2^2/2g$. Formula (30) of the last article correctly gives the lost head h' for this case, but when h_1 and h_2 are unknown it is convenient to express it in terms of v_1 and v_2 alone. The investigation given on page 180 of Treatise on Hydraulics (Ninth edition, 1912) shows that

$$h' = \frac{(v_1 - v_2)^2}{2g}$$

is the loss of head due to sudden expansion of section, or rather due to the sudden diminution of velocity which occurs in passing from the smaller to the larger section.

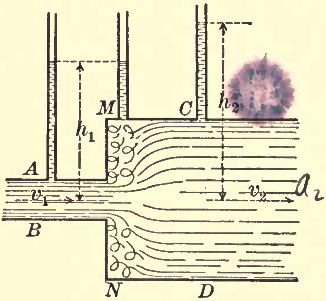


Fig. 34

Sudden enlargement of section should always be avoided in tubes and pipes owing to the loss of head that it causes, which may often be very great. For example, let there be no pressure-head in the section a_1 and let v_1 be due to a head h so that $v_1 = \sqrt{2gh}$; let the area a_2 be four times that of a_1 so that v_2 is one-fourth of v_1. The loss of head due to sudden expansion then is

$$h' = \frac{(v_1 - v_2)^2}{2g} = 9/16\, h$$

so that more than one-half of the energy of the water in a_1 is lost in impact, having been changed into heat. In the section a_2 the effective head is $7/16 h$, of which $1/16 h$ is velocity-head and $6/16 h$ is pressure-head.

When a sudden contraction of section in the direction of the flow occurs, as in Fig. 35, the water suffers a contraction similar to that in the standard orifice, and hence in its expansion to fill the second section a loss of head results. Let v_1 be the velocity in the larger section and

v that in the smaller, while v' is the velocity in the contracted section of the flowing stream; and let a_1, a and a' be the corresponding areas of the cross-sections. From the above formula the loss of head due to the expansion of section from a' to a is

$$h' = \frac{(v'-v)^2}{2g} = \left(\frac{a}{a'} - 1\right)^2 \frac{v^2}{2g} = \left(\frac{1}{c'} - 1\right)^2 \frac{v^2}{2g}$$

in which c' is the coefficient of contraction of the stream or the ratio of a' to a (Art. 24). The value of c' depends upon the ratio between the areas a and a_1. When a is small compared with a_1, the value of c' may be taken at 0.62 as for orifices (Art. 22). When a is equal to a_1, there is no contraction or expansion of the stream and c' is unity. Let d and d_1 be the diameters corresponding to the areas a and a_1, and let r be the ratio of d to d_1. Then experiments seem to indicate that c' is given by

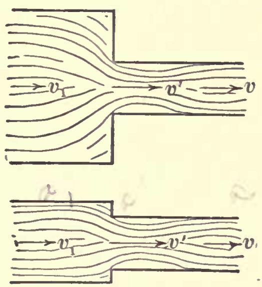


Fig. 35

$$c' = 0.582 + \frac{0.0418}{1.1 - r}$$

From this approximate values of c' can be computed,

for	r=0.0	0.4	0.6	0.7	0.8	0.9	0.95	1.0
	c'=0.62	0.64	0.67	0.69	0.72	0.79	0.86	1.00

Prob. 31 *A*. In a horizontal tube like Fig. 36 the diameters are 6 inches and 12 inches and the discharge is 1.57 cubic feet per second. Compute the loss of head due to the sudden enlargement.

Prob. 31 *B*. Compute the loss of head when a pipe which discharges 1.57 cubic feet per second suddenly diminishes in section from 12 to 6 inches diameter.

ART. 32. NOZZLES AND JETS

For fire service two forms of nozzles are in use. The smooth nozzle is essentially a conical tube like A in Fig. 28, the larger end being attached to a hose, but it is often provided with a cylindrical tip and sometimes the larger end is curved, as shown in Fig. 36. The ring nozzle is a

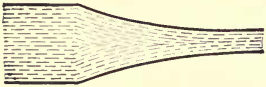


Fig. 36

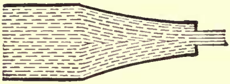


Fig. 37

similar tube, but its end is contracted so that the water issues through an orifice smaller than the end of the tube (Fig. 37). The experiments of Freeman show that the mean coefficient of discharge is about 0.97 for the smooth nozzle and about 0.74 for the ring nozzle. The smooth nozzle is used much more than the ring nozzle.

Let p be the pressure in pounds per square inch as found by a gage placed at the nozzle entrance, D the diameter in inches of the tip of the nozzle, d the diameter in inches of the pipe or hose at the nozzle entrance, and c the coefficient of discharge, then

$$q = 29.83\, D^2 \sqrt{\frac{p}{(1/c)^2 - (D/d)^4}}$$

gives the discharge in gallons per minute. For example, let $D=1$ inch $d=2.5$ inches $c=0.972$ for a smooth nozzle, $p=100$ pounds per square inch; then the formula gives $q=290$ gallons per minute.

For smooth nozzles the value of the coefficient of velocity c_1 is the same as that of the coefficient of discharge c, since the jet issues without contraction. The experiments of Freeman furnish the following mean values of the coefficient of discharge for smooth cone nozzles of different diameters under pressure-heads ranging from 45 to 180 feet

Diameter in inches =	¾	⅞	1	1⅛	1¼	1⅜
Coefficient c	=0.983	0.982	0.972	0.976	0.971	0.959

These values were determined by measuring the pressure p and the discharge q, from which c can be computed by the above formula. For example, a nozzle having a diameter of 1.001 inches at the end and 2.50 inches at the base discharged 208.5 gallons per minute under a pressure of 50 pounds per square inch at the entrance. Here $d_1=1.001$, $d=2.5$, $p=50$, and $q=208.5$, and inserting these in the formula and solving for c, there is found $c=0.985$.

In ring nozzles the ring which contracts the entrance is usually only 1/16 or ⅛ inch in width. The effect of this is to diminish the discharge, but the stream is sometimes thrown to a slightly greater height. On the whole, ring nozzles seem to have no advantage over smooth ones for fire purposes.

With pressures of 50 and 100 pounds per square inch at the entrance to smooth nozzles of one inch diameter, vertical jets rise to heights of about 101 and 152 feet in still air, but the limits of heights as good effective fire streams are about 73 and 96 feet respectively. The maximum horizontal distances to which the extreme drops from the jets can be thrown are about 160 and 230 feet, but the practical horizontal distances for effective fire streams are only about one-half these figures.

The ball nozzle, often used for sprinkling, has a cup at the end of the nozzle and within the cup a ball, so that the jet issuing from the tip of the nozzle is deflected sidewise in all directions. This apparatus exhibits a striking illustration of the principle of negative pressure, for the ball is not driven away from the tip, but is held close to it by the atmospheric pressure, the negative pressure-head being caused by the high velocity of the sheet of water around the ball. The cup is usually so arranged that the ball cannot be driven out of it, for this might occur under the first impact of the jet, but when the flow has become steady, there is no tendency of this kind, and the ball is seen slowly revolving upon the cushion of water without touching any part of the cup.

Prob. 32 *A*. When the coefficient of contraction of a jet is 1.00 and the coefficient of velocity of the nozzle is 0.97, what is the coefficient of discharge of the nozzle?

Prob. 32 *B*. Under a head of 6 feet the discharge from an orifice is 3.74 gallons per second. What head is necessary in order that the discharge may be one cubic foot per second?

Prob. 32 *C*. Compute the probable discharge through a vertical circular orifice of $\frac{3}{4}$ inches diameter under a head of 1.75 feet.

Prob. 32 *D*. An orifice one inch square in a gate like that of Fig. 8 is 3 feet below the higher water level and 2 feet below the lower water level. Compute the discharge in cubic feet per minute.

Prob. 32 *E*. When the coefficient of velocity is 0.98, compute the velocity from a nozzle of 1 inch diameter when attached to a hose of $2\frac{1}{2}$ inches diameter, the pressure at the entrance, as measured by a gage, being 43.4 pounds per square inch.

CHAPTER 4

FLOW THROUGH PIPES

ART. 33. FUNDAMENTAL IDEAS

Pipes made of clay were used in very early times for conveying water. The Romans used lead pipes for conveying water from their aqueducts to small reservoirs and from the latter to their houses. Frontinus gives a list of twenty-five standard sizes of pipes, varying in diameter from 0.9 to 9 inches, which were made by curving a sheet of lead about ten feet long and soldering the longitudinal joint. In modern times lead pipes have also been used for house service, but these are now largely superseded by either iron pipes or iron pipes lined with lead or tin. For the mains of city water supplies cast-iron pipes are most common, and since 1890 steel-riveted pipes have come into use for large sizes. Lap-welded wrought-iron or steel pipes are used in some cases where the pressure is very high, and large wooden stave pipes are in use in the western part of the United States. A very large pipe which brings water to a city is sometimes called an aqueduct, but the word is more properly applied to a brick or concrete conduit which is laid on a tolerably uniform grade and does not run full. For pipes, on the other hand, the section is fully filled and the water is often under pressure from high heads.

The phenomena of flow through a pipe are apparently simple. The water from the reservoir, as it enters the pipe, meets with more or less resistance, depending upon the manner of connecting, as in tubes (Art. 29). Resistances of friction and cohesion must then be overcome along the interior surface, so that the discharge at the end is much smaller than in the tube. When the flow becomes steady, the pipe is entirely filled throughout its length; and hence the mean velocity at any section is the same as that at the end when the size is uniform. This velocity is found to decrease as the length of the pipe increases, other things being equal, and becomes very small for great lengths, which shows that nearly all the head has been lost in overcoming the resistances. The length of the pipe is measured along its axis, following all the curves, if there be any. The velocity considered is the mean velocity, which is equal to the discharge divided by the area of the cross-section of the pipe.

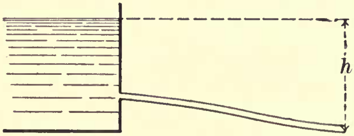


Fig. 38

The head which causes the flow is the difference in level from the surface of the water in the reservoir to the center of the end, when the discharge occurs freely into the air as in Fig. 38. If h be this head, and W the weight of water discharged per second, the theoretic potential energy per second is Wh; and if v be the actual mean velocity of discharge, the kinetic energy of the discharge is $W.v^2/2g$. The difference between these is the energy which has been transformed into heat in overcoming the resistances. Thus the total head is h, the velocity-head of the outflowing stream is $v^2/2g$, and the lost head is $h-v^2/2g$. If the lower end of the pipe is submerged, as

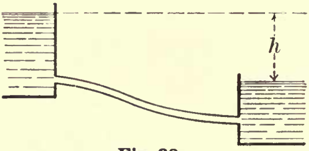


Fig. 39

in Fig. 39, the head h is the difference in elevation between the two water levels.

The total loss of head in a straight pipe of uniform size consists of two parts. First, there is a loss of head h' due to entrance, which is the same as in a short cylindrical tube, and secondly there is a loss of head h'' due to the frictional resistance of the interior surface. The loss of head at entrance is always less than the velocity-head and in this chapter it will be expressed by the formula

$$h' = m\frac{v^2}{2g}$$

From Art. 30 it is seen that the value of m is $(1/c_1)^2-1$ where c_1 is the coefficient of velocity. Hence for a pipe projecting into the reservoir (D in Fig. 29) m is 0.93, for a pipe with end flush with inner side of the reservoir m is 0.49, and for a perfect mouthpiece with rounded edges m is 0. When the condition of the end is not specified the value used for m in the following pages will be 0.5.

The loss of head in friction is very large for long pipes while the loss of head at entrance is very small. For example it is proved by actual gagings that a clean cast-iron pipe 10 000 feet long and 1 foot in diameter discharges about 4¼ cubic feet per second under a head of 100 feet. The mean velocity then is, if q be the discharge and a the area of the cross-section,

$$v = \frac{q}{a} = \frac{4.25}{0.7854} = 5.41 \text{ feet per second,}$$

and the probable loss of head at entrance hence is

$$h' = 0.5 \times 0.01555 \times 5.41^2 = 0.23 \text{ feet,}$$

or only one-fourth of one percent of the total head. In this case the effective velocity-head of the issuing stream is only 0.45 feet, which shows that the total loss of head is 99.55 feet, of which 99.32 feet are loss in friction.

Prob. 33 *A*. Under a head of 20 feet a pipe 1 inch in diameter and 100 feet long discharges 15 gallons per minute. Compute the loss of head at entrance.

Prob. 33 *B*. Consult Herschel's Water Supply of the City of Rome (Boston, 1899) and find several facts regarding the old aqueducts and pipes.

ART. 34. LOSS OF HEAD IN FRICTION

The loss of head due to the resisting friction of the interior surface of a pipe is usually large, and in long pipes it becomes very great, so that the discharge is only a small percentage of that due to the head. Many observations have been made upon pipes of different sizes and lengths under different velocities of flow, and the discussion of these has enabled the approximate laws to be deduced which govern the loss of head in friction. These laws are:

1. The loss of head in friction is directly proportional to the length of the pipe.
2. It is inversely proportional to the diameter of the pipe.
3. It increases nearly as the square of the velocity.
4. It is independent of the pressure of the water.
5. It increases with the roughness of the interior surface.

These five laws may be expressed by the formula

$$h'' = f\,\frac{l}{d}\,\frac{v^2}{2g} \tag{34}$$

in which l is the length of the pipe, d its diameter, f is an abstract number which depends upon the degree of roughness of the surface, and $v^2/2g$ is the velocity-head due to the mean velocity.

The quantity f may be called the friction factor, and its value ranges from 0.05 to 0.01 for new clean iron pipes. A mean value, often used in rough computations, is

$$\text{Friction factor } f = 0.02$$

The following table enables closer values of f to be selected for new clean cast-iron pipes when the diameter of the pipe and the mean velocity are known:

Vel. in feet per sec. =		1	2	3	4	6	10	15
For diameter	0.05 feet,	$f=0.047$	0.041	0.037	0.034	0.031	0.029	0.028
	0.1 feet,	$f=$ .038	.032	.030	.028	.026	.024	.023
	0.25 feet,	$f=$ .032	.028	.026	.025	.024	.022	.021
	0.5 feet,	$f=$ .028	.026	.025	.023	.022	.020	.019
	0.75 feet,	$f=$ .026	.025	.024	.022	.021	.019	.018
	1 foot,	$f=$ .025	.024	.023	.022	.020	.018	.017
	2 feet,	$f=$ .021	.020	.019	.017	.016	.014	.013
	3 feet,	$f=$ .019	.018	.016	.015	.014	.013	.012

To determine, therefore, the probable loss of head in friction, the velocity v must be known, and f is taken from the above table for the given diameter of pipes. The formula (34) then gives the probable loss of head in friction. For example, let $l = 10\,000$ feet, $d = 1$ foot, $v = 5.41$ feet per second. Then from the table the friction factor f is 0.021, and

$$h'' = 0.021 \times \frac{10{,}000}{1} \times 0.455 = 95.5 \text{ feet}$$

which is to be regarded as an approximate value, liable to an uncertainty of 5 percent.

Old pipes have interior surfaces much rougher than new ones, especially when tubercules are formed. The effect of these if often to render the friction factor f two or three times as large as the values given above.

✓ Prob. 34 *A*. Compute the loss of head in friction for a new cast-iron pipe 600 feet long and 3 inches in diameter, when its discharge is 2.5 cubic feet per minute.

ART. 35. VELOCITY AND DISCHARGE

The mean velocity in a straight pipe of uniform section can now be deduced. Let h be the total head and v the mean velocity in the pipe. The head h is equal to the head h' lost in entrance plus the head h'' lost in friction plus the velocity-head $v^2/2g$, or $h = h' + h'' + v^2/2g$. Inserting for h' its mean value from Art. 33 and for h'' the value given in Art. 34, and solving for v there is found

$$v = \sqrt{\frac{2gh}{1.5 + f\,(l/d)}}$$

which is a formula for the mean velocity. This formula applies only under such conditions that the entire pipe is filled with water; it cannot be used for a short vertical pipe discharging from a tank.

In this formula the friction factor f is a function of v to be taken from the table in Art. 34, and hence v cannot be directly computed, but must be obtained by successive approximations. For example, let it be required to compute the velocity of discharge from a pipe 1000 feet long and 6 inches in diameter under a head of 9 feet. Here $l = 1000$ feet $d = 0.5$ feet, $l/d = 2000$, $h = 9$ feet, and taking for f the rough mean value 0.02, the formula gives

$$v = \sqrt{\frac{2 \times 32.16 \times 9}{1.5 + 0.02 \times 2000}} = 3.7 \text{ feet per second}$$

The approximate velocity is hence 3.7 feet per second, and entering the table with this, the value of f is found to be 0.023. Then the formula gives

$$v = \sqrt{\frac{2 \times 32.16 \times 9}{1.5 + 0.023 \times 2000}} = 3.5 \text{ feet per second}$$

Again, entering the table with $v = 3.5$ and $d = 0.5$, there is found $f = 0.024$, and repeating the computation there is found $v=3.4$ feet per second. The conclusion therefore is that the velocity is about 3.4 feet per second if the pipe be new and clean.

The discharge per second from a pipe of given diameter is found by multiplying the velocity of discharge by the area of the cross-section of the pipe, or

$$q = \tfrac{1}{4}\pi d^2 v = 0.7854 d^2 v$$

in which v is to be found by the method described above.

For example, to find the discharge in gallons per minute from a clean pipe 3 inches in diameter and 1500 feet long under a head of 64 feet. Here $d=0.25$, $l=1500$, and $h=64$ feet. Then for $f=0.02$ the velocity is found to be 5.82 feet per second; then from the last article $f=0.024$ and the velocity becomes 5.30 feet per second. The discharge then is $q=0.7854 \times 0.25^2 \times 5.30 = 0.260$ cubic feet per second.

✓ Prob. 35 *A*. Compute the velocity in a new pipe 3 inches in diameter and 400 feet long under a head of 16 feet.

Prob. 35 *B*. Compute the discharge in cubic feet per second from a new pipe 0.25 feet in diameter and 400 feet long under a head of 16 feet.

ART. 36. LONG PIPES

A pipe is said to be long when $f(l/d)$ in the above formula for v is so large that the dropping of the number 1.5 in the denominator produces an error of less than one percent. This occurs, for the mean value of 0.02 for f, when l/d is greater than about 4000. For this case the formulas for velocity and discharge are

$$v = \sqrt{\frac{2ghd}{fl}} \qquad q = \tfrac{1}{4}\pi \sqrt{\frac{2ghd^5}{fl}}$$

which, for the English system of measures, become

$$v = 8.02\sqrt{\frac{hd}{fl}} \qquad q = 6.30\sqrt{\frac{hd^5}{fl}}$$

From these expressions for q the general and special formulas for computing the diameter of the pipe for a given discharge, length, and head are found to be

$$d^5 = \frac{8}{\pi^2 g}\frac{flq^2}{h} \qquad d = 0.479\left(\frac{flq^2}{h}\right)^{1/5}$$

In using these equations the friction factor f is taken from the table in Art. 34 for new clean pipes, and for old pipes from the best data obtainable (see Treatise on Hydraulics, Ninth Edition, Art. 106). In determining the proper size for a pipe which is to carry water liable to cause corrosion or tuberculation it is well to take f as about double of the values given in Art. 34.

For example, let it be required to determine the diameter of a new cast-iron pipe which is to deliver 500 gallons per second, its length being 4500 feet and the head 24 feet. Here the discharge is

$$q = 500/7.481 = 66.84 \text{ cubic feet per second.}$$

The approximate value of d then is

$$d = 0.479\left(\frac{0.02\times4500\times66.84^2}{24}\right)^{1/5} = 3.35 \text{ feet.}$$

From this the mean velocity of flow is

$$v = \frac{66.84}{0.7854\times3.35^2} = 7.6 \text{ feet per second,}$$

and from the table in Art. 37 the value of f for this diameter and velocity is found to be 0.013. Then the second computation gives $d = 3.08$ feet. The required diameter is therefore 3.1 feet, or about 37 inches; but as the regular market sizes of pipes furnish only 36 inches and 40 inches, one of these must be used, and it will be on the side of safety to select the larger.

If it is thought likely that this pipe may become so foul as to make f as high as 2×0.013, then the computation gives $d=3.54$ feet or about 42 inches. It is thus seen that the determination of the proper diameter for a new pipe involves elements which require the exercise of judgment trained by experience.

For circular orifices and for short tubes of equal length under the same head, the discharge varies as the square of the diameter. For pipes of equal length under a given head the discharges vary more rapidly owing to the influence of friction, since the above formula shows that if f be constant, q varies as $d^{5/2}$. The relative discharging capacities of pipes hence vary approximately as the $2\frac{1}{2}$ powers of their diameters. Thus, if two pipes of diameters d_1 and d_2 have same length and head, and if q_1 and q_2 be their discharges, then

$$q_1/q_2 = d_1^{5/2} d_2^{5/2} \quad \text{or} \quad q_2 = (d_2/d_1)^{5/2} q_1$$

For example, if there be two pipes of 6 and 12 inches diameter, d_2/d_1 equals 2 and hence $q_2=5.7q_1$, or the second pipe discharges nearly six times as much as the first. In a similar manner it can be shown that 32 pipes of 6 inches diameter have the same discharging capacity as 1 pipe 24 inches in diameter.

Prob. 36 *A*. Compute the discharge through a new pipe 9000 feet long and 36 inches diameter under a head of 48 feet.

Prob. 36 *B*. Compute the diameter required to deliver 15 000 cubic feet per hour through a new pipe 26 500 feet long under a head of 324.7 feet. If this quantity is carried in two pipes of equal diameter, what should be their size?

ART. 37. PIEZOMETERS

A piezometer is a vertical tube inserted into a water pipe. When the pipe is filled and there is no flow the water in the piezometer tube stands at the same level as that in the reservoir. When the flow occurs the water in the piezometer tube falls, and its height above the axis of the pipe shows the pressure-head which there exists.

Fig. 40 shows an inclined pipe, the two section areas a_1 and a_2 being unequal, so that the velocities therein are

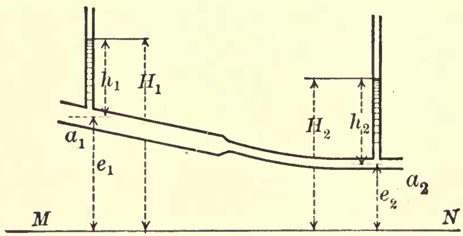


Fig. 40

v_1 and v_2, while the pressure-heads at these sections are h_1 and h_2. Let MN be any horizontal plane lower than the lowest section, as for instance the sea level, and let e_1 and e_2 be the elevations of a_1 and a_2 above it. With respect to this plane the weight W at a_1 has the potential energy We_1, the pressure-energy Wh_1, and the kinetic energy $W.v_1^2/2g$, or the total energy is

$$W(e_1+h_1+v_1^2/2g)$$

Similarly with respect to this plane the energy of W in a_2 is

$$W(e_2+h_2+v_2^2/2g)$$

If no losses of energy occur between the two sections, these expressions are equal, and hence

$$e_1+h_1+\frac{v_1^2}{2g}=e_2+h_2+\frac{v_2^2}{2g}$$

which is the general theorem of Bernouilli, namely:

> In any pipe, under steady flow without impact or friction, the gravity-head plus the pressure-head plus the velocity-head is a constant quantity for every section.

Now let H_1 and H_2 be the heights of the water levels in the piezometer tubes above the datum plane; then $e_1+h_1 = H_1$ and $e_2+h_2=H_2$, and accordingly

$$H_1+\frac{v_1^2}{2g}=H_2+\frac{v_2^2}{2g}$$

or, the piezometer elevation for a_1 plus the velocity-head is equal to the sum of the corresponding quantities for any other section.

The above theorem belongs to theoretical hydraulics, in which frictional resistances are not considered. Under actual conditions there is always a loss of energy or head, so that when water flows from a_1 to a_2, the first member of the above equation is larger than the second. Let Wh' be the loss in energy, then this is equal to the difference of the energies in a_1 and a_2 with respect to the datum plane, or,

$$h'=H_1-H_2+\frac{v_1^2}{2g}-\frac{v_2^2}{2g}$$

that is, the lost head is equal to the difference in level of the water surfaces in the piezometer tubes plus the differences of the velocity-heads.

The most common case is when the pipe is of uniform size throughout, or $a_1=a_2$, then v_1 and v_2 are equal because the flow is steady. The head lost in friction between any two sections is then simply,

$$h'=H_1-H_2$$

so that it is only necessary to ascertain the difference of elevation of the water in the piezometers by running a line of levels in order to determine the head lost in friction in the pipe between them.

Prob. 37 *A*. The water level in the piezometer at a_1 is 67.329 feet above a certain bench mark and that in the piezometer at a_2 63.791 feet above the same bench mark. The pipe being of uniform size, what is the loss of head in friction between the two sections?

Prob. 37 *B*. What is the loss of head for the above data when the section a_1 is 12 inches in diameter and that at a_2 is 9 inches in diameter, the discharge being 2.7 cubic feet per second?

ART. 38. THE HYDRAULIC GRADIENT

The hydraulic gradient is a line which connects the water levels in piezometers placed at intervals along the pipe; or rather, it is the line to which the water levels would rise if piezometer tubes were inserted. In Fig. 41 the line BC is the hydraulic gradient, and it is now to be shown that for a pipe of uniform size

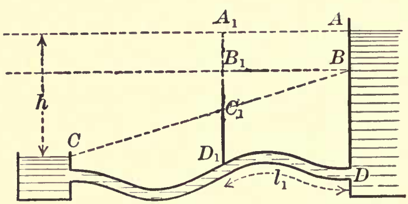


Fig. 41

this is approximately a straight line. For a pipe discharging freely into the air this line joins the outlet end with a point B near the top of the reservoir. For a pipe with submerged discharge, as in Fig. 41, it joins the lower water level with the point B.

Let D_1 be any point on the pipe distant l_1 from the reservoir, measured along the pipe line. The piezometer there placed rises to C_1, which is a point in the hydraulic gradient. The equation of this line with reference to the origin A is given by

$$H_1 = (1+m)\frac{v^2}{2g} + f\frac{l_1}{d}\frac{v^2}{2g}$$

in which H_1 is the ordinate A_1C_1, and l_1 is the abscissa AA_1, provided that the length of the pipe is sensibly equivalent to its horizontal projection. In this equation the first term of the second member is constant for a given velocity, and is represented in the figure by AB or A_1B_1; the second term varies with l_1, and is represented by B_1C_1. The gradient is therefore a straight line.

When there are easy horizontal curves in a pipe line, the above conclusions are unaffected, except that the gradient BC is always vertically above the pipe, and therefore can be called straight only by courtesy, although as before the ordinate B_1C_1 is proportional to l_1. When there are sharp curves, the inclination of the hydraulic gradient becomes greater and it is depressed at each curve by an amount equal to the loss of head which there occurs. When an obstruction occurs in a pipe, or a valve is partially closed, there is a sudden depression of the gradient at the obstruction or at the valve.

If a pipe is so laid that a portion of it rises above the hydraulic gradient as at D_1 in Fig. 42, an entire change of condition generally results. If the pipe is closed at C, all the piezometers stand in the line AA, at the same level as the surface of the reservoir. When the valve at C is opened, the flow at first occurs under normal conditions, h being the head and BC the hydraulic gradient.

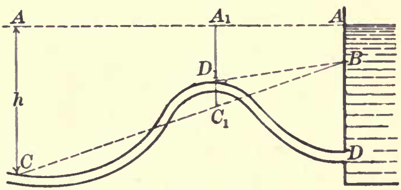


Fig. 42

The pressure-head at D_1 is then negative, as in a siphon (Art. 20), and is represented by D_1C_1. As a consequence air tends to enter the pipe, and when it does so, owing to defective joints, the continuity of the flow is broken, and then the pipe from D_1 to C is only partly filled with water. The hydraulic gradient is then shifted to BD_1, the discharge occurs at D_1 under the head A_1D_1, while the remainder of the pipe acts merely as a channel to deliver the flow. It usually happens that this change results in a great diminution of the discharge, so that it has been necessary to dig up and relay portions of a pipe line which have been inadvertently run above the hydraulic gradient. This trouble can always be avoided by preparing a profile of the proposed route, drawing the hydraulic gradient upon it, and excavating the pipe trench well below the gradient. In cases where the cost of this excavation is so great that it is resolved to lay the pipe above the gradient, all the joints of the pipe above the gradient should be made absolutely tight so that no air can enter the pipe and interrupt the flow.

Prob. 38 *A*. A pipe 3 inches in diameter discharges 538 cubic feet per hour under a head of 12 feet. What is the velocity in feet per second?

Prob. 38 *B*. At a distance of 300 feet from the reservoir the pipe in the last problem is 4.5 feet below the water surface of the reservoir. Compute the probable pressure-head at this point.

ART. 39. A COMPOUND PIPE

A compound pipe is one having different sizes in different portions of its length. The change from one length to another should be made by a "reducer," which is a conical frustrum several feet long, so that losses of head due to sudden enlargement or contraction are avoided. Let d_1, d_2, d_3, etc., be the diameters; l_1, l_2, l_3, etc., the corresponding lengths, the total length being l_1+l_2+ etc. Let v_1, v_2, etc., be the velocities in the different sections. Neglecting the loss of head at entrance and also that lost in curvature, the total head h may be placed equal to the loss of head in friction, or

$$h=f_1\frac{l_1}{d_1}\frac{v_1^2}{2g}+f_2\frac{l_2}{d_2}\frac{v_2^2}{2g}+\text{etc.}$$

Now if the steady discharge per second be q,

$$v_1=q/\tfrac{1}{4}\pi d_1^2 \qquad v_2=q/\tfrac{1}{4}\pi d_2^2 \qquad \text{etc.}$$

Substituting these velocities and solving for q, gives

$$q=\tfrac{1}{4}\pi\sqrt{\frac{2gh}{f_1\frac{l_1}{d_1^5}+f_2\frac{l_2}{d_2^5}+\text{etc.}}}$$

in which the friction factors f_1, f_2, etc., corresponding to the given diameters and computed velocities are found from the table in Art. 34.

For example, consider the case of a pipe having only two sizes; let $d_1=2$ and $l_1=2800$ feet, $d_2=1.5$ and $l_2=2145$ feet, and $h=127.5$ feet. Using for f_1 and f_2 the mean value, 0.02, and making the substitutions in the formula, there is found $q=26.2$ cubic feet per second, from which $v_1=8.3$ and $v_2=14.8$ feet per second. Now it is found that $f_1=0.015$ and $f_2=0.015$; and repeating the computation, $q=30.2$ cubic feet per second, whence $v_1=9.6$ and $v_2=17.1$ feet per second. These results are probably as definite as the table of friction factors will allow, but are to be regarded as liable to an uncertainty of five percent or more.

A compound pipe may be used to prevent the hydraulic gradient from falling below the pipe line. Thus, it is seen in Fig. 43 that the hydraulic gradient rises at D_1

and falls at D_2, and that its slope over the larger pipe is less than over the smaller one. These slopes and the amount of rise at D_1 can be computed for a given case. Using the above numerical data, the loss of head in friction is by Art. 34 for 100 feet of the large pipe $h''=1.07$ feet, while the same for the small pipe is 4.55 feet. Hence the slope of the gradients AC_1 and C_2C is more than four times as rapid as that of the gradient E_1E_2. In the large pipe at D_1 the velocity-head is $0.01555\times 9.6^2=1.43$ feet, and, supposing that no loss occurs in the reducer, the velocity-head for the small pipe is 4.55 feet. The vertical rise C_1E_1 of the hydraulic gradient at D_1 is hence the rise in pressure-head $4.55-1.43=3.12$ feet, and a fall of equal amount occurs at D_2.

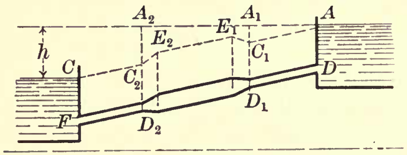


Fig. 43

Prob. 39. Near Rochester, N. Y., there is a pipe 102 277 feet long, of which 50 828 feet is 36 inches in diameter and 51 449 feet is 24 inches in diameter. Under a head of 143.8 feet this pipe is said to have discharged in 1876 about 14 cubic feet per second and in 1890 about 10½ cubic feet per second. Compute the discharge by the above formula, using the friction factors for riveted pipe which are given in Art. 45.

omit

ART. 40. HOUSE-SERVICE PIPES

A service pipe which runs from a street main B (Fig. 44) to a house is connected to the former at right angles, and usually by a corporation cock or by a "ferrule." The loss of head at entrance in such cases is hence larger than in those before discussed, and m should probably be taken as at least equal to unity. The pipe, if of lead, is frequently carried around sharp corners by curves of small radius; if of iron, these curves are formed by pieces forming a quadrant of a circle into which the straight parts are screwed, each curve causing a loss of head nearly equal to double the velocity-head.

A water main should be so designed that a certain minimum pressure-head h_1 exists in it at times of heaviest draft. This pressure-head may be represented by the height of the piezometer column AB, which would rise in a tube supposed to be inserted in the main, as in Fig. 44. The head h which causes the flow in the pipe is then the difference in level between the top of this column and the end of the pipe, or AC. Inserting for h this value, the formulas of Art. 38 may be applied to the investigation of service pipes in the manner there illustrated.

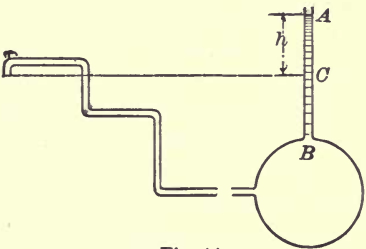


Fig. 44

Since the sizes of common house-service pipes are regulated by the practice of the plumbers and by the market sizes obtainable, it is not often necessary to make computations regarding the flow of water through them.

The detection and prevention of the waste of water by consumers is a matter of importance in cities where the supply is limited and where meters are not in use. Of the many methods devised to detect this waste, one by the use of piezometers may be noticed, by which an inspector without entering a house may ascertain whether water is being drawn within, and the approximate amount per second. Let M in Fig. 45 be the street main from which a service pipe MOH runs to a house H. At the edge of the sidewalk a tube OP is connected to the service pipe, which has a three-way cock at O, which can be turned from above. The inspector, passing on his rounds in the night-time, attaches a pressure gage at P and turns the cock O so as to shut off the water from the house and allow the full pressure of the main p_1 to be registered. Then he turns the cock so that the water may flow into the house, while it also rises in OP and registers the pressure p_2. Then if p_2 is less than p_1, it is certain that waste is occurring in the house, and notice thereof is given to the consumer.

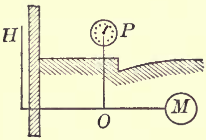


Fig. 45

When the pressure in the street main is very high, a pressure regulator may be placed between the main and the house in order to reduce the pressure and thus allow lighter pipes to be used in the house. Fig. 46 shows the principle of its action, where A represents the pipe from the main and B the pipe leading to the house. A weight W is placed upon a piston which covers the opening into the chamber C. This weight and that of the piston are sufficient to overcome a certain unit-pressure in C, and therefore the unit-pressure in B is less than that in A by that amount. For example, suppose the pressure in A to be 100 pounds per square inch, and let it be required that the pressure in B shall not rise above 60 pounds per square inch; then the piston must be so weighted that it may exert on the water in C a pressure

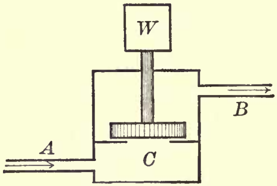


Fig. 46

of 40 pounds per square inch. When water is drawn out anywhere along the pipe B, the pressure in the chamber above the piston falls below 60 pounds per square inch, and hence the piston rises and water flows from A into B until the pressure is restored. Instead of a weight, a spring is generally used, or sometimes a weighted lever.

Prob. 40 *A*. When the pressure in the main is 80 pounds per square inch, what is the pressure-head in feet?

Prob. 40 *B*. When the pressure in the main is 80 pounds per square inch, what is the pressure in the bath room when there is no flow, the bath room being 36 feet higher than the main?

Art. 41. Steel and Wood Pipes

All the preceding principles and formulas apply to pipes of any kind, but the friction factors in Art. 34 are for new clean cast-iron pipes. The diameters of such pipes are rarely greater than 4 feet, and the largest ever used are 5 feet.

Pipes 36 inches and larger in diameter have been made of wrought-iron or steel plates riveted together. Wrought-iron, however, is now but little used, on account of its higher cost, except in the form of thin sheets for temporary pipes. Each section usually consists of a single plate which is bent into the circular form and the edges united by a longitudinal riveted lap joint. The different sections are then riveted together in transverse joints so as to form a continuous pipe. At AB (Fig. 47) is shown the so-called taper joint, where the end of each section goes into the end of the following one, as in a stovepipe, the flow occurring in the direction from A to B. At CD is seen the method of cylinder joints where the sections are alternately larger and smaller. For the large sizes double rows of rivets are used both in the longitudinal and transverse joints, the style of riveted joint depending on the pressure of water to be carried by the pipe. Riveted pipes have also been built with butt

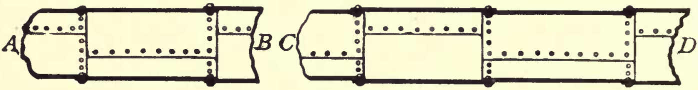


Fig. 47

joints on both longitudinal and transverse seams, lap plates being on the outside.

Owing to the friction caused by the rivets and joints the discharge from riveted pipes is less than that from cast-iron pipes in which the obstruction caused by the joints is very slight. The following values of the friction factor f, which have been derived from the data given by Herschel, are applicable to new clean riveted pipes coated with asphaltum in the usual manner.

Velocity in feet per second,		$v =$	1	2	3	4	5
Cylinder joints	3 ft. diam.,	$f =$	0.035	0.029	0.024	0.021	0.019
	4 ft. diam.,	$f =$	.025	.022	.020	.020	.021
Taper joints	$3\frac{1}{2}$ ft. diam.,	$f =$	.025	.024	.023	.022	.022
	4 ft. diam.,	$f =$	.027	.026	.025	.024	.023

These friction factors are approximately double those given for new cast-iron pipes in Art. 90, this increase being largely due to the friction of the rivet heads and lapped joints, though some of it is probably chargeable to the roughness of the asphaltum coating. It must be noted that these factors increase with age.

Wood pipes were used in several American cities during the years 1750–1850, these being made of logs laid end to end, a 3 or 4 inch hole having been first bored through each log. Pipes formed of redwood staves were first used in California about 1880, these staves being held in place by bands of wrought-iron arranged so that they could be tightened by a nut and screw. Several long lines of these large pipes have been built in the Rocky Mountains and Pacific States. They have also been used there for city mains to a limited extent.

Gagings of a wood pipe 6 feet in diameter made by Marx, Wing, and Hoskins furnish values of the friction factor f for velocities ranging from 1 to 5 feet per second,

Velocity in feet per second,	$v=$	1	2	3	4	5
1897,	$f=$	0.026	0.019	0.017	0.016	
1899,	$f=$	.019	.018	.017	.017	.017

These values show that this wood pipe became smoother after two years' use, while a steel pipe becomes rougher.

Prob. 41. Compute the discharge in gallons per day from a taper jointed riveted steel pipe 2 miles long and 42 inches in diameter when the slope of the hydraulic gradient is 17.5 feet per mile.

ART. 42. A PIPE WITH A NOZZLE

Water is often delivered through a nozzle in order to perform work upon a motor or for the purposes of hydraulic mining, the nozzle being attached to the end of a pipe which brings the flow from a reservoir. In such a case it is desirable that the pressure at the entrance to the nozzle should be as great as possible, and this will be effected when the loss of head in the pipe is as small as possible. The pressure column in a piezometer supposed to be

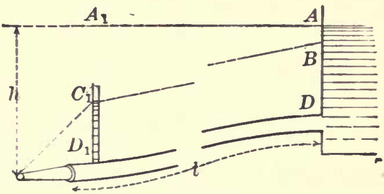


Fig. 48

inserted at the end of the pipe, as shown at C_1D_1 in Fig. 48, measures the pressure-head there acting, and the height A_1C_1 measures the lost head plus the velocity-head, the latter being very small.

Let h be the total head on the end of the nozzle, D its diameter, and V the velocity of the issuing stream. Let d and v be the corresponding quantit es for the pipe, and l its length. Then the effective velocity-head of the issuing stream is $V^2/2g$, and the lost head is $h-V^2/2g$. This lost head consists of that lost at the entrance, that lost in friction in the pipe, and lastly, that lost in the nozzle. Then the principle of energy gives the equation

$$h-\frac{V^2}{2g}=m\frac{v^2}{2g}+f\frac{l}{d}\frac{v^2}{2g}+m'\frac{V^2}{2g}$$

Here m is determined by Art. 33, f by Art. 34, while m' for the nozzle is found in the same manner as m is found for the pipe, or $m'=(1/c_1)^2-1$, where c_1 is the coefficient of velocity for the nozzle (Art. 30). This value of m' takes account of all losses of head in the nozzle, so that it is unnecessary to consider its length; for a perfect nozzle c_1 is unity and m' is zero.

The velocities v and V are inversely as the areas of the corresponding cross-sections (Art. 20), since the flow is steady, whence $V=v(d/D)^2$. Inserting this in the above equation and solving for v gives

$$v=\sqrt{\frac{2gh}{m+f(l/d)+(1/c_1)^2(d/D)^4}}$$

for the velocity in the pipe. The velocity and discharge from the nozzle are then found by

$$V=(d/D)^2v \qquad q=\tfrac{1}{4}\pi D^2V$$

and the velocity-head of the jet is $V^2/2g$. These equations show that the greatest value of V obtains when D is as small as possible compared to d, and that the greatest discharge occurs when D is equal to d. When the object of a nozzle is to utilize the velocity-head of a jet, a large pipe and a small nozzle should be employed.

As a numerical example, the effect of attaching a nozzle to the pipe whose discharge was computed in Art. 35 will be considered. There $l=1500$, $d=0.25$, and $h=64$ feet; $m=0.5$, $v=5.3$ feet per second, and $q=0.26$ cubic feet per second. Now let the nozzle be one inch in diameter at the small end, or $D=0.0833$ feet, and let its coefficient c_1 be 0.98. Here $d/D=3$, and for $f=0.025$ the velocity in the pipe is

$$v=\sqrt{\frac{2\times 32.16\times 64}{0.5+0.025\times 1500\times 4+1.041\times 81}}$$

or $v=4.2$ feet per second. The effect of the nozzle, therefore, is to reduce the velocity in the pipe. The velocity of the jet at the end of the nozzle is, however,

$$V=v(d/D)^2=37.8 \text{ feet per second,}$$

and the discharge per second from the nozzle is

$$q=\tfrac{1}{4}\pi D^2 V=0.206 \text{ cubic feet}$$

which is about 20 percent less than that of the pipe before the nozzle was attached. The nozzle, however, produces a marvelous effect in increasing the energy of the discharge; for the velocity-head corresponding to 5.3 feet per second is only 0.44 feet, while that corresponding to 37.8 feet per second is 22.2 feet, or about 50 times as great. As the total head is 64 feet, the efficiency of the pipe and nozzle is about 35 percent.

Fire hose is generally 2½ inches in diameter. The experiments of Freeman show that the friction factor f ranges from 0.038 to 0.034 for unlined linen hose, from 0.030 to 0.029 for rough rubber-lined cotton hose, and from 0.024 to 0.018 for smooth rubber-lined cotton hose. By using these values the above formulas are directly applicable to fire hose.

Prob. 42 *A*. What head is required to discharge 3 gallons per minute through a new pipe 1 inch in diameter and 1000 feet long?

Prob. 42 *B*. Compute the diameter of a new pipe to deliver 50 gallons per minute under a head of 4 feet when the length is 5000 feet.

Prob. 42 *C*. How many 12-inch pipes are required to deliver the same quantity of water as a pipe 60 inches in diameter?

Prob. 42 *D*. Which pipe will carry the greater quantity of water, a cast-iron pipe 42 inches in diameter or a steel riveted pipe of 40 inches diameter?

Prob. 42 *E*. A 2½ inch unlined cotton hose is 480 feet long and has a 1-inch smooth nozzle at its end. Compute the discharge in gallons per minute when the pressure at the hydrant or steamer is 100 pounds per square inch.

To here

CHAPTER 5

FLOW IN CONDUITS AND RIVERS

ART. 43. DEFINITIONS

From the earliest times water has been conveyed from place to place in artificial channels, such as troughs, aqueducts, ditches, and canals, there being no head to cause the flow except that due to the slope. The Roman aqueducts were usually rectangular channels about $2\frac{1}{2}$ feet wide and 5 feet deep, lined with cement, sometimes running underground and sometimes supported on arches. The word "conduit" will be used as a general term for a channel of any shape lined with timber, mortar, or masonry, and will also include troughs, sewers, and large pipes. Conduits may be either open, as in the case of troughs, or closed, as in sewers and most aqueducts. Ditches and canals are conduits in earth without artificial lining. Most of the principles relating to conduits and canals apply also to streams, and the word "channel" will be used as applicable to all cases.

The wetted perimeter of the cross-section of a channel is that part of its boundary which is in contact with the water. Thus, if a circular sewer of diameter d be half full of water, the wetted perimeter is $\frac{1}{2}\pi d$. In this chapter the letter p will designate the wetted perimeter.

The hydraulic radius of a water cross-section is its area divided by its wetted perimeter, and the letter r will be used to designate it. If a is the area of the cross-section, the hydraulic radius of that section is found by $r = a/p$. The letter r is of frequent occurrence in formulas for the flow in channels; it is a linear quantity which is always expressed in the same unit as p, and hence its numerical value is different in different systems of measures. It is frequently called the hydraulic depth or hydraulic mean depth, because for a shallow section its value is but little less than the mean depth of the water. Thus, in Fig. 49, if b be the breadth on the water surface, the mean depth is a/b, and the hydraulic radius is a/p; and these are nearly equal, since the length of p is but slightly larger than that of b. The hydraulic radius of a circular cross-section filled with water is one-fourth of the diameter; thus

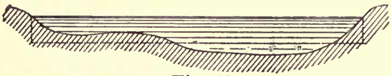


Fig. 49

$$r = a/p = \tfrac{1}{4}\pi d^2/\pi d = \tfrac{1}{4}d$$

The same value is also applicable to a circular section half filled with water, since then both area and wetted perimeter are one-half their former values.

The slope of the water surface in the longitudinal section, designated by the letter s, is the ratio of the fall h to the length l in which that fall occurs, or $s = h/l$. The slope is hence expressed as an abstract number, which is independent of the system of measures employed. To determine its value with precision h must be obtained by referring the water level at each end of the line to a bench-mark by the help of a hook gage or other accurate means, the benches being connected by level lines run with care. The distance l is not measured horizontally but along the inclined channel, and it should be of considerable length in order that the relative error in h may not be large. If $s = 0$ there is no slope and no flow; but when there is even the smallest slope the force of gravity furnishes a component acting down the inclined surface, and motion ensues.

The flow in a channel is said to be steady when the same quantity of water per second passes through each cross-section. Let a_1, a_2, a_3, etc., be the areas at several cross-sections and v_1, v_2, etc., be the mean velocities, then

$$q = a_1 v_1 = a_2 v_2 = a_3 v_3$$

and when the discharge is known the mean velocities are $v_1 = q/a_1$, $v_2 = q/a_2$, etc. The definition of mean velocity hence is that it is a velocity which multiplied by the area of the cross-section gives the discharge, or $v = q/a$.

Prob. 43 *A*. Compute the hydraulic radius of a rectangular trough whose width is 5.6 feet and depth 2.8 feet.

Prob. 43 *B*. The elevations of the water surface at two points 2.786 miles apart on a river are 627.318 and 642.407 feet. Compute the slope s if the same is uniform.

ART. 44. FORMULA FOR MEAN VELOCITY

When all the wetted cross-sections of a channel are equal, and the water is neither rising nor falling, having attained the condition of steady flow, the flow is said to be uniform. This is the case in a conduit or canal of constant size and slope whose supply does not vary. The same quantity of water per second then passes each cross-section, and consequently the mean velocity in each section is the same. This uniformity of flow is due to the resistances along the interior surface of the channel, for were it perfectly smooth the force of gravity would cause the velocity to be accelerated. The entire energy of the water due to the fall h is hence expended in overcoming resistances caused by surface roughness.

Let W be the weight of water passing any cross-section in one second, F the force of friction per square unit along the surface, p the wetted perimeter, and h the fall in the length l. The potential energy of the fall is Wh. The total resisting friction is Fpl, and the energy consumed per second is $Fplv$, if v be the velocity. Accordingly $Fplv$ equals Wh But the value of W is wav, if w is the weight of a cubic foot of water and a the area of the cross-section in square feet. Hence $Fpl=wah$, and since a/p is the hydraulic radius r, and h/l is the slope s, this reduces to $F=wrs$, which is an approximate expression for the resisting force of friction on one square unit of the surface of the channel. In order to establish a formula for the mean velocity the value of F must be expressed in terms of v, and this can only be done from the results of experiments which indicate that F is approximately proportional to the square of the mean velocity. Therefore, if c is a constant, the mean velocity

$$v=\mathrm{c}\sqrt{rs} \tag{44}$$

is which is the formula of Chezy. This is really an empirical expression, since the relation between F and v is derived from experiments.

Another method of establishing Chezy's formula for channels is to consider that when a pipe on a uniform slope is not under pressure, the hydraulic gradient coincides with the water surface. Then formula (44) may be used by replac ng h'' by h and d by its value $4r$. Accordingly

$$h=\tfrac{1}{4}f\frac{l}{r}\frac{v^2}{2g} \quad \text{or} \quad v=\sqrt{8g/f}\sqrt{rs}$$

in which the quantity $\sqrt{8g/f}$ is the Chezy coefficient c.

This coefficient c is different in different systems of measures since it depends upon g. For the English system it is found that c usually lies between 30 and 160, and that its value varies with the hydraulic radius and the slope, as well as with the roughness of the surface. To determine the value of c for a particular case the quantities v, r, and s are measured, and then c is computed. To determine v the flow must be gaged either in a measuring vessel or by an orifice or weir, or, if the channel be large, by floats or other indirect methods described in the next chapter, and then the mean velocity v is computed from $v=q/a$. It being a matter of great importance to establ'sh a satisfactory formula for mean velocity, thousands of such gagings have been made, from which values of the coefficients have been deduced.

Prob. 44. Compute the value of c for a circular masonry conduit 6 feet in diameter which delivers 65 cubic feet per second when running half full, its slope or grade being 1.5 feet in 1000 feet.

ART. 45. CIRCULAR CONDUITS

Circular conduits are large wood, steel, clay, and cement pipes, and sewers and aqueducts of small size which are laid in place in the trench. When such a conduit of diameter d runs either full or half-full of water, the hydraulic radius r is $\frac{1}{4}d$, and the Chezy formula for mean velocity and the discharge are

$$v = c\sqrt{rs} \qquad q = av$$

in which r always has the value $\frac{1}{4}d$ and a is either the area of the circular section or one-half that area, as the case may be.

The following values of c are for new conduits having quite smooth interior surfaces and no sharp bends:

Velocity in feet per second	= 1	2	3	4	6	10
For diameter 3 feet,	c = 117	124	128	133	136	143
For diameter 4 feet,	c = 123	130	134	137	142	150
For diameter 5 feet,	c = 128	134	139	142	147	155
For diameter 6 feet,	c = 132	138	142	145	150	
For diameter 8 feet,	c = 137	143	148	151		

To use this table a tentative method must generally be employed, since c depends upon the velocity of flow. For this purpose there may be taken roughly

$$\text{mean Chezy coefficient } c = 125$$

and then v may be computed for the given diameter and slope; a new value of c is then taken from the table and a new v computed; and thus, after two or three trials, the probable mean velocity of flow is obtained.

For example, let it be required to find the velocity and discharge of a semicircular conduit of 6 feet diameter when laid on a grade of 0.1 foot in 100 feet. Here $r=$ 6/4 feet and

$$v = 125 \times \tfrac{1}{2}\sqrt{6 \times 0.001} = 4.8 \text{ feet per second.}$$

For this velocity the table gives 147 for c; hence

$$v = 147 \times \tfrac{1}{2}\sqrt{0.006} = 5.7 \text{ feet per second.}$$

Again, from the table $c = 150$, and

$$v = 150 \times \tfrac{1}{2}\sqrt{0.006} = 5.8 \text{ feet per second.}$$

This shows that 150 is a little too large; for $c = 149.5$, v is found to be 5.79 feet per second, which is the final result. The discharge per second now is

$$q = 0.7854 \times \tfrac{1}{2} \times 36 \times 5.79 = 81.9 \text{ cubic feet,}$$

which is the probable flow under the given conditions.

To find the diameter of a circular conduit to discharge a given quantity under a given slope, the area a is to be expressed in terms of d in the above equation, which is then to be solved for d; thus

$$d = \left(\frac{8q}{\pi c\sqrt{s}}\right)^{2/5} \qquad d = \left(\frac{16q}{\pi c\sqrt{s}}\right)^{2/5}$$

the first being for a conduit running full and the second for one running half full. Here c may be at first taken as 125; then d is computed, the approximate velocity found from $v=q/\frac{1}{4}\pi d^2$, and with this value of v a value of c is selected from the table, and the computation for d is repeated.

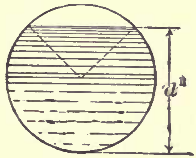


Fig. 50

When a circular conduit of diameter d is filled to the depth d' (Fig. 50), the wetted perimeter is

$$p=\tfrac{1}{2}\pi d+d \text{ arc sin } \frac{2d'-d}{d}$$

and the sectional area of the water surface is

$$a=\tfrac{1}{4}dp+(d'-\tfrac{1}{2}d)\sqrt{d'(d-d')}$$

From these p and a can be computed, and then r is found by dividing a by p. For any slope s the velocity v is the greatest when r has its largest value and the discharge q is the greatest when $a\sqrt{r}$ has its largest value. It can be shown that the velocity is the greatest when $d'=0.81d$ and that the discharge is the greatest when $d'=0.95d$; also that the discharge is 5 percent greater when $d'=0.95d$ than when the conduit is entirely filled.

Prob. 45. Compute the diameter of a circular conduit for a discharge of 81.9 cubic feet per second, the conduit running full and the slope being 1 foot in 1000 feet.

ART. 46. RECTANGULAR CONDUITS

In designing an open rectangular trough or conduit to carry water there is a certain ratio of breadth to depth which is most advantageous, because thereby either the discharge is the greatest or the least amount of material is required for its construction. Let b be the breadth and d the depth of the water section, then the area a is bd and the wetted perimeter p is $b+2d$. If the area a is given, it may be required to find the relation between b and d so that the discharge may be a maximum. If the wetted perimeter p is given, the relation between b and d to produce the same result may be demanded. In both cases it can be shown that the breadth is double the depth, or $b=2d$. This is called the most advantageous proportion for an open rectangular conduit.

The velocity and discharge through a rectangular conduit are expressed by the general equations

$$v=\mathrm{c}\sqrt{rs} \qquad\qquad q=av=\mathrm{c}a\sqrt{rs}$$

and are computed without difficulty for any given case when the coefficient c is known. To determine this, however, is not easy, for it is only from recorded experiments that its value can be ascertained. When the depth of the water in the conduit is one-half of its width, thus giving the most advantageous section, the values of c for smooth interior surfaces may be estimated by the use of the table for circular conduits, although c is probably smaller for rectangles than for circles of equal area.

Prob. 46 *A*. Compare the discharge of a trough 1×3 feet with that of two troughs each 1×2 feet.

Prob. 46 *B*. Find the size of a trough, whose width is double its depth, which will deliver 125 cubic feet per minute when its slope is 0.002, taking the coefficient c as 100.

ART. 47. KUTTER'S FORMULA

An elaborate discussion of all recorded gagings of channels was made by Ganguillet and Kutter in 1869, from which an important empirical formula was deduced for the coefficient c in the Chezy formula $v = c\sqrt{rs}$. The value of c is expressed in terms of the hydraulic radius r, the slope s, and the degree of roughness of the surface, and may be computed when these three quantities are given. When r is in feet and v in feet per second, Kutter's formula for the Chezy coefficient c is

$$c = \frac{\dfrac{1.811}{n} + 41.65 + \dfrac{0.00281}{s}}{1 + \dfrac{n}{\sqrt{r}}\left(41.65 + \dfrac{0.00281}{s}\right)}$$

in which n is an abstract number whose value depends only upon the roughness of the surface. By inserting this value of c in the Chezy formula for v, the mean velocity is made to depend upon r, s, and the roughness of the surface. The following values of n were assigned by Kutter to different surfaces:

$n = 0.009$ for well-planed timber,
$n = 0.010$ for neat cement,
$n = 0.011$ for cement with one-third sand,
$n = 0.012$ for unplaned timber,
$n = 0.013$ for ashlar and brick work,
$n = 0.015$ for unclean surfaces in sewers and conduits,
$n = 0.017$ for rubble masonry,
$n = 0.020$ for canals in very firm gravel,
$n = 0.025$ for canals and rivers free from stones and weeds,
$n = 0.030$ for canals and rivers with some stones and weeds,
$n = 0.035$ for canals and rivers in bad order.

The formula of Kutter has received a wide acceptance on account of its application to all kinds of surfaces. Notwithstanding that it is purely empirical, it is to be regarded as a formula of great value, so that no design for a conduit or channel should be completed without employing it in the investigation, even if the final construction be not based upon it. In sewer work it is extensively employed, n being taken as about 0.015. The formula shows that the coefficient c always increases with r, that it decreases with s when r is greater than 3.28 feet, and that it increases with s when r is less than 3.28 feet. When r equals 3.28 feet, the value of c is simply $1.811/n$.

Extended tables showing the values of c for different values of r and s may be found in the larger treatises on hydraulics and also in the American Civil Engineer's Pocket Book. But for a single case there is no difficulty in directly computing it from the above formula. For example, take a rectangular conduit lined with neat cement, $b=5.94$ feet, $d=0.91$ feet, $s=0.0049$. Here $n=0.010$, and $r=0.697$ feet. Inserting all values in the formula, there is found $c=148$.

Prob. 47. Compute the value of c for a rectangular trough of unplaned plank, 3.93 feet wide, in which the water is 1.29 feet deep, the slope being 0.49 feet in 100 feet.

ART. 48. DITCHES AND CANALS

Ditches for irrigating purposes are of a trapezoidal section, and the slope is determined by the fall between the point from which the water is taken and the place of delivery. If the fall is large, it may not be possible to construct the ditch in a straight line between the two points, even if the topography of the country should permit, on account of the high velocity which would result. A velocity exceeding 2 feet per second may often injure the bed of the channel by scouring, unless it be protected by riprap or other lining.

The principles of the preceding articles are sufficient to solve all usual problems of uniform flow in such channels when the values of the Chezy coefficient c are known.

These are best determined by Kutter's formula. As an example, let it be required to find the discharge for the case of Fig. 51, the longitudinal slope s being 0.001, the bottom width being 4.5 feet, the depth d being 5.5 feet, the side slope θ being 45°, and the channel being in tolerably good order so that n in Kutter's formula is 0.025. Here the top width is 15.5 feet, the section area is 55.0 square feet, the wetted perimeter is 20.0 feet, and the hydraulic radius is 2.75 feet. The value of c as computed from Kutter's formula is 71. Then the velocity is $v = c\sqrt{rs} = 3.7$ feet per second and the discharge through the ditch is $q = av = 203$ cubic feet per second. This velocity is so high that a very firm bed is required to resist it, since a velocity of 2 feet per second on the bed moves gravel and one of 3 feet per second moves pebbles one inch in size. Hence if this problem is one of design the bottom width should be increased and the depth of water be decreased in order to diminish the velocity and keep the discharge at about 200 cubic feet per second.

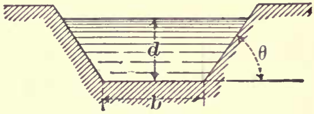


Fig. 51

A canal is a large ditch to which the above principles and methods directly apply. For a navigation canal the velocity should be quite small so that boats may not be retarded when running against the current.

Prob. 48. A canal of trapezoidal section is 60 feet wide on the top, the depth of water is 6 feet, and the sides make angles of 30° with the horizontal. Compute the wetted perimeter, the section area, and the hydraulic radius.

ART. 49. STREAMS AND RIVERS

Steady flow in a river channel occurs when the same quantity of water passes each section in each unit of time;

here the mean velocities in different sections vary inversely as the areas of those sections. Uniform flow is that particular case of steady flow where the sections considered are equal in area. Non-steady flow occurs when the stage of a river is rising or falling.

No branch of hydraulics has received more detailed investigation than that of the flow in river channels, and yet the subject is but imperfectly understood. The great object of all these investigations has been to devise a simple method of determining the mean velocity and discharge without the necessity of expensive field operations. In general it may be said that this end has not yet been attained, even for the case of uniform flow. Of the various formulas proposed to represent the relation of mean velocity to the hydraulic radius and the slope, none has proved to be of general practical value except the empirical one of Chezy given in the last chapter, and this is often inapplicable on account of the difficulty of measuring the slope s and determining the coefficient c. The fundamental equations for discussing the laws of variation in the mean velocity v and the discharge q are

$$v = \mathrm{c}\sqrt{rs} \qquad\qquad q = a.\mathrm{c}\sqrt{rs}$$

where a is the area of the cross-section and r its hydraulic radius, and all the general principles of the last chapter are to be taken as directly applicable to uniform flow in natural channels. Kutter's formula for c (Art. 44) is probably the best in the present state of science, although it is now generally recognized that it gives too large values for small slopes. In using it the roughness factors for rivers in good condition may be taken from Art. 44, but for bad regimen n is to be taken at 0.03, and for violent streams at 0.04 or greater.

When these formulas are used to determine the discharge of a river, a long straight portion or reach should be selected where the cross-sections are as nearly as possible uniform in shape and size. The width of the stream is then divided into a number of parts and soundings taken at each point of division. The data are thus obtained for computing the area a and the wetted perimeter p, from which the hydraulic depth r is derived. To determine the slope s a length l is to be measured, at each end of which bench-marks are established whose difference of elevation is found by precise levels. The elevations of the water surfaces below these benches are then to be simultaneously taken, whence the fall h in the distance l becomes known. As this fall is often small, it is very important that every precaution be taken to avoid error in the measurements, and that a number of them be taken in order to secure a precise mean.

Prob. 49. Which has the greater discharge, a stream 2 feet deep and 80 feet wide on a slope of 1 foot per mile, or a stream 3 feet deep and 40 feet wide on a slope of 2 feet per mile?

ART. 50. TRANSPORTING CAPACITY

It is well known that the water of rapid streams transports large quantities of earthy matter, either in suspension or by rolling it along the bed of the channel. It is now to be shown that the diameters of bodies which can be moved by the pressure of a current vary as the square of its velocity, and that their weights vary as the sixth power of the velocity.

When water causes sand or pebbles to roll along the bed of a channel, it must exert a force approximately proportional to the square of the velocity and to the area exposed (Art. 18), or if d is the diameter of the body and C a constant, the force which is required to move it horizontally is $F = Cd^2v^2$. But if motion just occurs, this force is also proportional to the weight of the body, because the frictional resistance of one body upon another varies as the normal pressure or weight. And as the weight of a sphere varies as the cube of the diameter, it follows that $d^3 = Cd^2v^2$, or $d = Cv^2$. Now since d varies as v^2, the weight of the body, which is proportional to d^3, must vary as v_6; which proves the proposition enunciated above. Hence an increase in velocity causes far greater increase in transporting capacity.

Since the weight of sand and stones when immersed in water is only about one-half their weight in air, the frictional resistances to their motion are slight, and this helps to explain the circumstance that they are so easily transported by currents of moderate velocity. It is found by observation that a pebble about one inch in diameter is rolled along the bed of a channel when the velocity is about 3½ feet per second; hence, according to the above theoretical deduction, a velocity five times as great, or 17½ feet per second, will carry along stones of 25 inches diameter. The following gives velocities in feet per second which are required to move the materials stated.

	Bottom velocity	Mean velocity
Clay fit for pottery,	0.3	0.4
Sand, size of anise seed,	0.4	0.5
Gravel, size of peas,	0.6	0.8
Gravel, size of beans,	1.2	1.6
Shingle, about 1 inch in diameter,	2.5	3.5
Angular stones, about 1½ inches,	3.5	4.5

The general conclusion to be derived from these figures is that ordinary small, loose earthy materials will be transported or rolled along the bed of a channel by velocities of 2 or 3 feet per second. It is not necessarily to be inferred that this movement of the materials is of an injurious nature in streams with a fixed regimen, but in artificial canals the subject is one that demands close attention. The velocity of the moving objects after starting has been found to be usually less than half that of the current.

Prob. 50. In the early history of the earth the moon was half its present distance from the earth's center, and the tides were about eight times as high as at present. It is supposed that these tides rolled over the low lands and moved great rocks from place to place. The highest velocity of such a wave is $\sqrt{gd}$, where d is the depth of the water. What is the probable weight and size of the largest rock that such a current would move?

Art. 51. Steady Non-Uniform Flow

When the cross-sections of a stream vary in size, as is generally the case, and the same quantity of water passes through each section in the same time, the condition called non-uniform flow arises. The surface slope of the water is here not parallel to the bed when the bed has a uniform slope, but is usually either concave or convex to the bed. These two cases are shown in Figs. 52 and 53, where *BB*

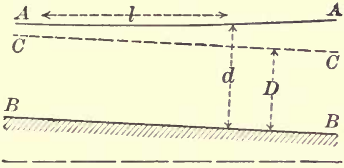


Fig. 52

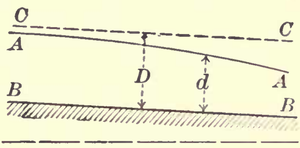


Fig. 53

is the bed of the stream which has the uniform slope i and *CC* is the slope of the water surface when the steady flow is uniform.

The backwater curve shown in Fig. 52 occurs when a dam or other obstruction exists downstream which has the effect of raising the water level, then the water surface rises to *AA* and the surface curve becomes tangent to *CC* when d equals D, which may be a long distance upstream. The backwater curve is a fruitful source of litigation because it always extends upstream a much greater distance than is at first supposed. The down-drop curve shown in Fig. 53 occurs when water is drawn out of a stream or canal for power or irrigation. Here the curve is concave to the bed and it becomes tangent to the original surface CC only at a long distance up stream.

At any point the inclination of the surface curve to the bed BB is given by

$$s=\frac{g}{c^2}\,\frac{v^2-c^2di}{v^2-gd}$$

where d is the mean depth of the water, i the slope of the bed, v the mean velocity, c the Chezy coefficient, and g the acceleration of gravity. When v is equal to $c\sqrt{di}$, then $s=0$, and the slope s is parallel to the bed of the stream. When v is less than $c\sqrt{di}$ the backwater curve of Fig. 52 results. When v is greater than $c\sqrt{di}$ the drop down curve of Fig. 53 results.

A very curious phenomenon which sometimes occurs in shallow channels is that of the so-called "jump," as shown in Fig. 54. This happens when the denominator in the above formula is zero; then s is infinite, and the water surface stands normal to the bed. Placing that denominator equal to zero, there is

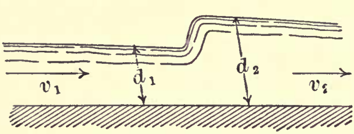


Fig. 54

found $v_2=gd$. Above the jump where the depth is d_1 the velocity is slightly greater than $\sqrt{gd_1}$, and below it is less than $\sqrt{gd_2}$. The slope i of the bed must be greater than g/c^2 in order that the jump may occur. The height d_2-d_1 ranges from twice to four times the depth d_1 and is rarely as large as one or two feet.

A more striking case of a vertical water front is seen when a large body of water moves down a cañon after a heavy rainfall, or when a reservoir bursts and allows a large discharge to suddenly escape down a narrow valley (Fig. 55). In the great flood of 1889 at Johnstown, Pa., such a vertical wall of water, variously estimated at from 10 to 30 feet in height, moved down the valley, carrying on its front brush and logs mingled with spray and foam. In 41 minutes it traveled a distance of 13 miles down the descent of 380 feet; the velocity was hence about 28 feet per second. The theoretic velocity being $v = \sqrt{gd}$, the value of d found from this equation is 24 feet.

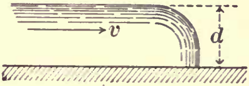


Fig. 55

The tidal bore, which occurs in many large rivers when the tide flows in at their mouths, obeys similar laws. The great bore at Hangchow, China, which occurs twice a year, is said to travel up the river at a rate of from 10 to 13 miles per hour, the height of the vertical front being from 10 to 20 feet. From $v = \sqrt{gh}$, the velocity corresponding to a depth of 10 feet is 12.6 miles per hour, while that corresponding to a depth of 20 feet is 17 miles per hour, so that the statements have a fair agreement with the theoretical law.

Prob. 51 *A*. Bidone, who was the first to make experiments on the jump, found for $v_1 = 5.59$ feet per second and $d_1 = 0.208$ feet, that the depth d_2 was 0.613 feet. The jump formula being $d_2 = 2\sqrt{d_1 v_1^2/2g}$, compute the theoretic depth d_2.

Prob. 51 *B*. A stone weighing 0.5 pounds is moved by a current of 3 feet per second; what is the weight of a stone which will be moved by a current of 9 feet per second?

CHAPTER 6

MEASUREMENT OF WATER

ART. 52. DIRECT METHODS

Some of the most important practical problems of Hydraulics are those of the measurement of the amount of water discharged in one second by an orifice, pipe, conduit, or river. When the discharge is quite small a very precise way is to catch it in a barrel which is set on the platform of a weighing scale and thus ascertain the weight which is delivered in a given time, from which by Art. 3 the total volume can be computed, and then the discharge per second is equal to this volume divided by the number of seconds elapsed during the experiment. A larger quantity may be measured in a rectangular tank, the section area of which is accurately known; here the height of the water surface is noted at the beginning and end of the experiment, and the volume is then found by multiplying the area by the difference of the two heights.

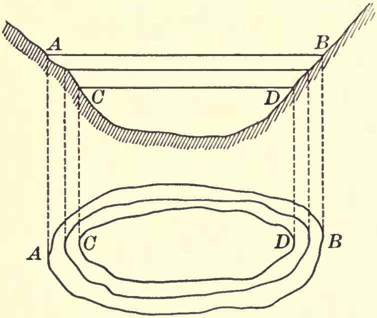


Fig. 56

Large quantities of water are sometimes measured in the reservoir of a city supply. A precise contour map of the reservoir being made (Fig. 56), the volume between successive contour planes is computed. Then as the water enters the height of the surface is determined at regular intervals of time, and from these data the discharge into the reservoir can be computed.

The height of the water level may be read on a fixed scale board, or a glass gage tube may be set upon which the height of the water surface above the orifice can be read at any time during the experiment (Fig. 57). Another method is to have a float on the water surface, the vertical motion of which is communicated to a cord passing over a pulley, so that readings can be taken on a scale as the weight at the lower end of the cord moves up or down. For very many cases, however, these methods are not sufficiently precise, and the hook gage is then used. A hook gage consists of a rod at the lower end of which is a hook which can be raised or lowered until its sharp point is at the water level (see Fig. 60). Just before the point of the hook pierces the skin of the water (Art. 2) a pimple or protuberance is seen to rise above it. The hook is supported by a graduated rod upon which readings can be taken, by help of a vernier, to thousandths of a foot.

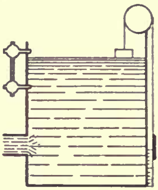


Fig. 57

Another method of gaging is by making measurements on the velocity of the water, from which and the known section areas the discharge can be computed. The principal ways of measuring the velocities of flowing water will be briefly described in the following pages. While this method is only partially direct it is one which must necessarily be used when large quantities are involved.

Water measurement by means of weirs is an indirect method, since it depends upon coefficients which have been established through direct gagings. But such coefficients are so much more accurately known than those of orifices, conduits, and streams that the errors in them may be regarded as very small. Moreover, weirs may be used for both small and moderately large quantities of water and when once built the expense of taking the necessary readings is not great. Weirs are generally used in tests of hydraulic motors for measuring the water that flows through them.

The miner's inch is a unit of water measurement used in mining operations in the western part of the United States. It may be roughly defined to be the quantity of water which will flow from a vertical standard orifice one inch square, when the head on the center of the orifice is $6\frac{1}{2}$ inches. From Art. 26 the coefficient of discharge is seen to be about 0.623 and accordingly the actual discharge from the orifice in cubic feet per second is $q = \frac{1}{144} \times 0.623 \times 8.02 \sqrt{6.5/12} = 0.0255$ and the discharge in one minute is $60 \times 0.255 = 1.53$ cubic feet. The mean value of one miner's inch is therefore about 1.5 cubic feet per minute. The actual value, however, varies in different states. In California 40 miner's inches make one cubic foot per minute, in Colorado 38.4 is the equivalent, while in other states it is 50. The miner's inch as a unit for water measurement is awkward and confusing, and it is greatly to be desired the cubic foot per second should always be used.

Prob. 52 *A*. Water flows from an orifice uniformly for 89.3 seconds and falls into a barrel on a platform weighing scale. The weight of the empty barrel is 27 pounds and that of the barrel and water is 276 pounds. What is the discharge of the orifice in gallons per minute, when the temperature of the water is 62° Fahrenheit?

Prob. 52 *B*. Let the areas within the contour curves *AB* and *CD* in Fig. 56 be 84 320 and 79 624 square feet, their vertical distance apart being 5 feet. Also let the area of the contour half-wav between AB and CD be 82 150 square feet. Compute the volume in cubic feet included between AB and CD.

ART. 53. STANDARD WEIRS

A weir is a notch in the top of the vertical side of a vessel or reservoir through which water flows. The notch is generally rectangular, and the word "weir" will be used to designate a rectangular notch unless otherwise specified, the lower edge of the rectangle being truly horizontal, and its sides vertical. The lower edge of the rectangle is called the "crest" of the weir. In Fig. 58 is shown the outline of the most usual form, where the vertical edges of the notch are sufficiently removed from the sides of the reservoir or feeding canal, so that the sides of the stream may be fully contracted; this is called a weir with end contractions. In the form of Fig. 59 the edges of the notch are coincident with the sides of the feeding canal, so that the filaments of water along the sides pass over without being deflected from the vertical

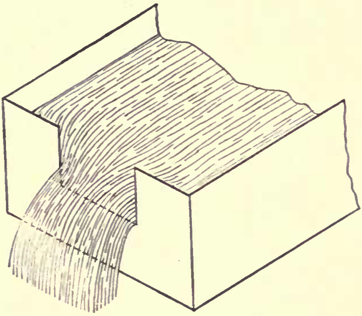


Fig. 58

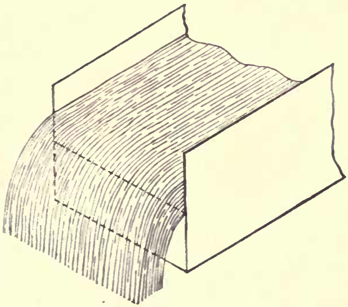


Fig. 59

planes in which they move; this is called a weir without end contractions, or with end contractions suppressed.

It is necessary in order to make accurate measurements of discharge by a weir that the same precaution should be taken as for orifices (Art. 21); namely, that the inner edge of the notch shall be a definite angular corner so that the water in flowing out may touch the crest only in a line, thus insuring complete contraction, as in Fig. 60. In precise observations a thin metal plate will be used for a crest, while in common work it may be sufficient to have the crest formed by a plank of smooth hard wood with its inner corner cut to a sharp right angle.

Weirs are extensively used for measuring the discharge of small streams, and for determining the quantity of water supplied to hydraulic motors; the practical importance of the subject is so great that numerous experiments have been made to ascertain the laws of flow, and the coefficients of discharge. Since the head on the crest of a weir is small, it must be determined with precision in order to avoid error in the computed discharge. The hook gage, seen in Fig. 60, is generally used for accurate work. For rough gagings of streams the heads may be determined by setting a post a few feet upstream from the weir and on the same level as the crest, and measuring the depth of the water over the top of the post by a scale graduated to tenths and hundredths of a foot.

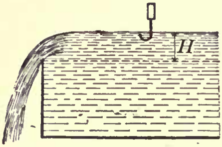


Fig. 60

The head H on the crest of the weir is in all cases to be measured several feet upstream from the crest, as indicated in Fig. 60. This is necessary because of the curve taken by the surface of the water in approaching the weir. The distance to which this curve extends back from the crest of the weir depends upon many circumstances, but it is generally considered that perfectly level water will be found at 2 or 3 feet back of the crest for small weirs, and at 6 or 8 feet for very large weirs. It is desirable that the hook should be placed at least one foot from the sides of the feeding canal, if possible.

Prob. 53 *A*. If a feeding canal 3.5 feet wide discharges 12 cubic feet per second when the water is 2.1 feet deep, what is the mean velocity of flow?

Prob. 53 *B*. A hook gage reads 0.207 feet when its point is at the level of the crest of a weir. What is the head H when the water level is read as 0.631 feet?

ART. 54. DISCHARGE OVER A WEIR

For a weir with end contractions (Fig. 58) which has the length b and on whose crest the head is H (Fig. 60), the theoretic discharge is found from the second formula of Art. 26 by making $h_1=0$ and $h_2=H$. Then

$$Q=\tfrac{2}{3}\sqrt{2g}\,.\,bH^{3/2}$$

and introducing the coefficient of discharge c,

$$q=c\,.\,\tfrac{2}{3}\sqrt{2g}\,.\,bH^{3/2} \tag{54}$$

is the formula for computing the actual discharge.

The following table gives values of the coefficient c to be used for contracted weir (Fig. 58) in the above formula.

Length of crest in feet, $b=$		0.66	1	2	3	5	10
For $H=0.1$ feet,	$c=$	0.632	0.639	0.646	0.652	0.653	0.656
For $H=0.15$ feet,	$c=$	.619	.625	.634	.638	.640	.641
For $H=0.2$ feet,	$c=$	.611	.618	.626	.630	.631	.633
For $H=0.25$ feet,	$c=$	.605	.612	.621	.624	.626	.628
For $H=0.3$ feet,	$c=$	.601	.608	.616	.619	.621	.624
For $H=0.4$ feet,	$c=$	.595	.601	.609	.613	.615	.618
For $H=0.6$ feet,	$c=$	.587	.593	.601	.605	.608	.613

As an example of the use of the formula and table, let it be required to find the discharge per second over a contracted weir 4 feet long when the head H is 0.457 feet. From the table the coefficient of discharge is 0.614 for $H=0.4$ and 0.6065 for $H=0.6$, which gives about 0.612 when $H=0.457$. Then the discharge per second is

$$q=0.612\times\tfrac{2}{3}\times 8.02\times 4\times 0.457^{3/2}=4.04 \text{ cubic feet.}$$

Weirs without end contractions (Fig. 59) have a discharge from 1 to 6 percent greater than contracted weirs, the greatest differences being when the length b is short. For very long weirs the discharges of the two kind of weirs are practically equal. For precise measurements of discharge, weirs with end contractions are always preferred, and velocity of approach is to be taken into account in the computation in the manner described in Merriman's Treatise on Hydraulics.

Prob. 54 *A*. Find from the table the coefficient of discharge for a contracted weir 2.5 feet long, when the head on the crest is 0.237 feet.

Prob. 54 *B*. Compute the discharge for a contracted weir 5 feet long when the head on the crest is 0.268 feet.

ART. 55. OTHER WEIR FORMULAS

J. B. Francis made in 1854 extensive experiments on weirs and deduced the following formulas for discharge. These formulas, although derived from large weirs, are extensively used, when approximate results are alone required, for weirs of length greater than 4 feet and heads greater than 0.4 feet. The length b and the head H being expressed in feet, and the discharge in cubic feet per second, the formula for weirs with end contractions is

$$q=3.33(b-0.2H)H^{3/2}$$

and that for weirs without end contractions is

$$q=3.33bH^{3/2}$$

Here the number 3.33 is $\frac{2}{3}c\sqrt{2g}$ where c is the true coefficient of discharge. The 88 experiments from which this number was deduced show that the coefficient 3.33 actually ranged from 3.30 to 3.36, so that the use of the formula may perhaps give an error of one percent in the computed discharge.

A waste weir is a rectangular notch in the top of a dam which allows the surplus water to escape. Waste weirs usually have wide and rounded crests. The formula

$$q=\text{M}bH^{3/2}$$

is employed for these, the value of M ranging from 2.5 to 4.0. When the crest is narrow and the front vertical M is 3.33, as in Francis' formula. When the crest is about 3 feet wide and level, with an inclined approach back of it, M is about 3.01. This formula is also used for finding the approximate discharge of a stream across which a dam has been built. For this purpose the proper coefficient must be selected from records of experiments which are found in more extended treatises on hydraulics.

Triangular weirs are sometimes used for the measurement of small quantities of water, the arrangement being shown in Fig. 61. Such a weir must have sharp inner corners, so that the stream may be fully contracted, and the sides should have equal slopes, and the angle at the vertex should be a right angle. The depth of water above this lower vertex is to be measured by a hook gage in the usual manner at a point several

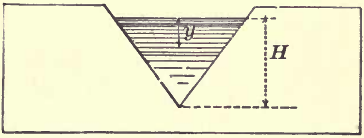


Fig. 61

feet upstream from the notch. The formula for the discharge is $q=2.53H^{5/2}$ in which H must be in feet and q is in cubic feet per second.

Trapezoidal weirs (Fig. 62) are sometimes used instead of rectangular ones. The slope of the sides should be 4 on 1 or $z/H=1/4$, and the other arrangements as in the weir with end contractions. The formula for the discharge is $q=3.367bH^{3/2}$ in which b is the length of the crest in feet, H being in feet and q in cubic feet per second.

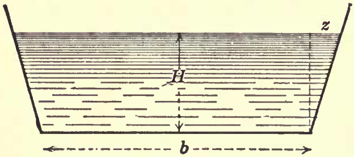


Fig. 62

Prob. 55 *A*. For a weir with end contractions $b=7$ feet and $H=0.457$ feet. Compute the discharge by Francis' formulas.

Prob. 55 *B*. Compute the discharges through a triangular weir when $H=1$ foot and $H=2$ feet.

ART. 56. WATER METERS

Meters used for measuring the quantity of water supplied to a house or factory are of the displacement type; that is, as the water passes through the meter it displaces or moves a piston, a wheel, or a valve, the motion of which is communicated through a train of clock wheels to dials where the quantity that has passed since a certain time is registered. There is no theoretical way of determining whether or not the readings of the dial hands are correct, but each meter must be rated by measuring the discharge in a tank. Several meters may be placed on the same pipe line in this operation, the same discharge then passing through each of them. When impure water passes through a meter for any length of time, deposits are liable to impair the accuracy of its readings, and hence it should be rerated at intervals.

The piston meter is one in which the motion of the water causes two pistons to move in opposite directions, the water leaving and entering the cylinders by ports which are opened and closed by slide valves somewhat similar to those used in the steam-engine. The rotary meter has a wheel enclosed in a case so that it is caused to revolve as the water pases through. The screw meter has an encased helical surface that revolves on its axis as the water enters at one end and passes out at the other. The disk meter has a wabbling disk so arranged that its motion is communicated to a pin which moves in a circle. In all these, and in many other forms, it is intended that the motion given to the pointers on the dials shall be proportional to the volume of water passing through the meter. The dials may be arranged to read either cubic feet or gallons, as may be required by the consumers.

The Venturi meter, named after the distinguished hydraulician who first experimented on the principle by which it operates, was invented by Herschel in 1887. Fig. 63 shows a horizontal pipe having an area a_1 at each

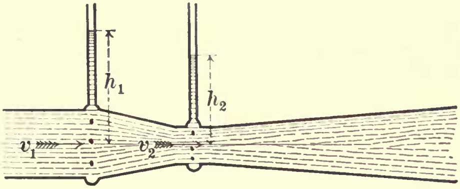


Fig. 63

end, and the central part contracted to the area a_2, with two small piezometer tubes into which the water rises. When there is no flow, the water stands at the same level in these two columns, but when it is in motion, the heights of these columns above the axis of the pipe are h_1 and h_2. Let v_1 and v_2 be the mean velocities in the two cross-sections. Then by Art. 24 the effective head in the upper section is $h_1+v_1^2/2g$, and that in the small section is $h_2+v_2^2/2g$; if there be no losses caused by friction, these two expressions must be equal, and hence by the theorem of Art. 20, $v_2^2-v_1^2=2g(h_1-h_2)$. Now let Q be the discharge through the pipe, or $Q=a_1v_1$ and also $Q=a_2v_2$. Taking the values of v_1 and v_2 from these, inserting them in the above equation, and solving for Q, gives

$$Q=\frac{a_1a_2}{\sqrt{a_1^2-a_2^2}}\sqrt{2g(h_1-h_2)}$$

which may be called the theoretic discharge. Owing to frictional losses which occur between the two cross-sections, the actual discharge q is always less than Q, or $q=cQ$, in which c is a coefficient whose value generally lies between 0.95 and 0.99.

The two water columns shown in Fig. 63 may be led to a mercury gage where the difference h_1-h_2 is shown by the difference in level of the mercury columns. But usually in practise an automatic recording apparatus is employed, a pointer carrying a pen being moved as the difference h_1-h_2 varies, so that the actual discharge q at every instant is recorded on a paper dial. The Venturi meter is used for measuring the discharge through pipes two inches or more in diameter, the largest meters of this type yet undertaken being those for the new Catskill water system of the city of New York. Each of these meters has a capacity of 650 000 000 gallons per day, the diameter of each end of the meter tube being 210 inches, while that at the contracted section is 93 inches.

Prob. 56. A 12-inch pipe delivers 810 gallons per minute through a Venturi meter, a_2 being one-ninth of a_1. Compute the mean velocities in the sections a_1 and a_2. If the pressure-head in a_1 is 21.4 feet, compute the pressure-head in a_2.

ART. 57. THE PITOT TUBE

About 1750 the French hydraulic engineer Pitot invented a device for measuring the velocity in a stream by means of the velocity-head which it will produce. In its simplest form it consists of a bent tube, the mouth of which is placed so as to directly face the current (Fig. 64). The water then rises in the vertical part of the tube

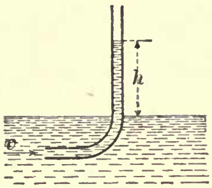


Fig. 64

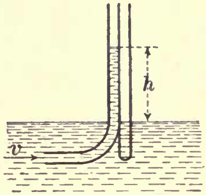


Fig. 65

to a height h above the surface of the flowing stream, and this height is equal to the velocity-head $v^2/2g$, so that the actual velocity v is in practice approximately equal to $\sqrt{2gh}$. As constructed for use in streams, Pitot's apparatus consists of two tubes placed side by side with their submerged mouths at right angles, so that when one is opposed to the current, as seen in Fig. 65, the other stands normal to it, and the water surface in the latter tube hence is at the same level as that of the stream. Both tubes are provided with cocks which may be closed while the instrument is immersed, and it can be then lifted from the water and the head h be read at leisure. It is found that the actual velocity is always less than $\sqrt{2gh}$, and that a coefficient must be deduced for each instrument by moving it in still water at known velocities.

In 1888 Freeman made experiments on the distribution of velocities in jets from nozzles, in which an improved form of Pitot tube was used. The point of the tube facing the current was the tip of a stylographic pen, the diameter of the opening being about 0.006 inch. This point was introduced into different parts of the jet and the pressure caused in the tube was measured by a Bourdon pressure gage reading to single pounds. The velocities of the jets were high, often over 50 feet per second. He concluded that any velocity as determined by the tube was smaller than that computed from $v = \sqrt{2gh}$ by less than one percent. This investigation established the fact that the Pitot tube is an instrument of great precision for the measurement of high velocities.

Prob. 57. Consult Engineering News, May 4, 1911, and ascertain how a Pitot tube may be used to determine the speed of a boat or ship.

ART. 58. THE CURRENT METER

In 1790 the German hydraulic engineer Woltmann invented an apparatus for measuring the velocity of flowing water which was later improved and is now extensively used for gaging streams and other open channels. This meter is like a windmill, having three or more vanes mounted on a spindle and so arranged that the face of the wheel always stands normal to the direction of the current, the pressure of which causes it to revolve. The number of revolutions of the wheel is approximately proportional to the velocity of the current. In the best forms of this instrument the number of revolutions made in a given time is determined and recorded by an apparatus placed near the observer on a bridge, in a boat, or elsewhere. In these forms an electric connection is made and broken at every fifth revolution and a dial on the recording apparatus affected. By means of a telephone receiver the making and breaking of the circuit can be made audible to the observer, who in such case simply keeps count of the number of clicks and observes on a stop-watch the time elapsed during a given number of revolutions.

A current meter cannot be used for determining the velocity in a small trough or channel, since the introduction of it into the cross-section would contract the area and cause a change in the velocity of the flowing water. In large conduits, canals, and rivers it is, however, a convenient and accurate instrument. By simply holding it at a fixed position below the surface, the velocity at that point is found; by causing it to descend at a uniform rate from surface to bottom, the mean velocity in that vertical is obtained; and by passing it at a uniform rate over all parts of the cross-section of a channel, the mean velocity v can be directly determined. For velocities greater than 5 feet per second, the use of the current meter is very difficult.

To derive the velocity of the water from the number of recorded revolutions per second the current meter must first be rated by placing it on a car which is pushed at a known velocity through still water. The best place for doing this is in a pond or navigation canal, where the water has no sensible velocity. It is found that the velocity of the car is not exactly proportional to the recorded number of revolutions, but is usually expressed by the equation $V=a+bn$ where V is the velocity and n the number of revolutions per second, and a and b are constants which differ for different conditions.

Prob. 58. For velocities of 0.7 and 4.2 feet per second the number of revolutions per second of the wheel of a current meter were found to be 0.5 and 2.0. Find the constants a and b in the equation $V=a+bn$.

ART. 59. GAGING OF STREAMS

For any orifice, tube, conduit, or stream let a be the area of the cross-section and v the uniform velocity, then the discharge is $q=av$; hence, if a and v can be found by measurement, q is known. In fact, however, the velocity varies in different parts of a cross-section, so that the determination of v cannot be directly made. Yet there always is a certain value for v, which multiplied into a will give the actual discharge q, and this value is called the mean velocity.

In the case of a stream or open channel the velocity is much less along the sides and bottom than near the middle. A rough determination of the mean velocity may be made, however, by observing the greatest surface velocity by a float, and taking eight-tenths of this for the approximate mean velocity. Thus, if the float requires 50 seconds to run 120 feet, the mean velocity is about 1.9 feet per second; then if the cross-section be 820 square feet, the discharge is 1560 cubic feet per second.

A common method of finding the discharge of a stream is to subdivide the cross-section into parts and determine their areas a_1, a_2, etc., the sum of which is the total area a (Fig. 66). Then, if v_1, v_2, etc., are the mean velocities in these areas as determined by observations, the discharge of the stream is

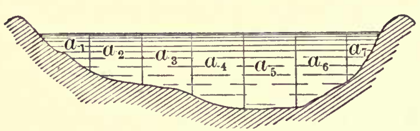


Fig. 66

$$q=a_1v_1+a_2v_2+a_3v_3+\text{etc.}$$

Here the mean velocities may be roughly found by observing the passage of a surface float at the middle of each subdivision and multiplying this surface velocity by

0.86. By the use of a current meter, passing it down and up through the middle of each subdivision, much closer values may be determined. When q has been computed, the mean velocity v may be derived from $v = q/a$.

Gages for reading the stages of water are now set up on many rivers, and daily observations are taken. Such a gage is usually a vertical board graduated to feet and tenths and set if possible with its zero below the lowest known water level. Another form is the box-and-chain gage, which consists of a box fastened on a bridge with a graduated scale within it and a chain that can be let down to the water level; the length of the chain being known, the gage height can then be read from the scale. Such observations of the daily stage of a river are of great value in planning engineering constructions, and they are now made at many stations by the United States government. An approximate rule for the variation in velocity and discharge with small changes in depth is the following: when the mean depth changes 1 percent the mean velocity changes 0.5 percent and the discharge changes 1.5 percent.

When several measurements of the discharge of a stream have been made for different stages of water, a

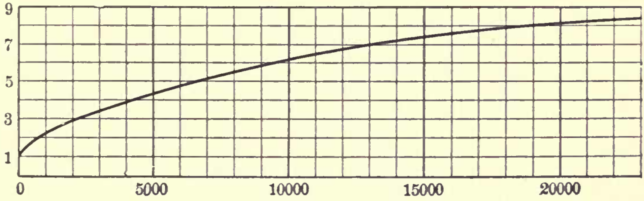


Fig. 67

curve may be drawn to show the law of variation of discharge, and from this curve the discharge corresponding to any given stage of water may be approximately ascertained. Fig. 67 shows the actual discharge curve for the Lehigh River at Bethlehem, Pa., the ordinates being the heights of the water level as read on the gage, and the abscissas being the discharges of the river in cubic feet per second. Each station on a river has its own distinctive discharge curve, for the local topography influences the heights to which the water level will rise.

Prob. 59 *A*. A stream of 4 feet mean depth delivers 800 cubic feet per second. What will be the discharge when the depth is decreased to 3.87 feet; what will be the velocity when the depth is 4.12 feet?

Prob. 59 *B*. A circular conduit pipe, 3 feet in diameter, is divided into three parts by concentric circles whose diameters are 1 and 2 feet. The mean velocities in the three parts are found to be 6.6, 4.8, and 3.0 feet per second. Compute the discharge and mean velocity.

ART. 60. VELOCITIES IN A CROSS SECTION

By means of the Pitot tube and the current meter velocities in different parts of many cross-sections have been measured. At the contracted section of a jet from an orifice all the velocities are found to be equal to the velocity due to the head. For a pipe the greatest velocity is found to be at the center and the least along the circumference of the cross-section, while the mean velocity is about 84 percent of that at the center. Hence a Pitot tube with its tip at the center of the pipe will determine a fair value of the mean velocity.

For a conduit or aqueduct running partly full, the greatest velocity is near the middle of the section but some distance below the surface, while the smallest velocities are at the sides and along the bottom. Diagrams showing the distribution for large aqueducts may be seen in Merriman's Treatise on Hydraulics, Ninth Edition, page 310.

In Fig. 68 there is shown at A a cross-section of a stream with contour curves of equal velocity; here the greatest velocity is seen to be near the deepest part of the section

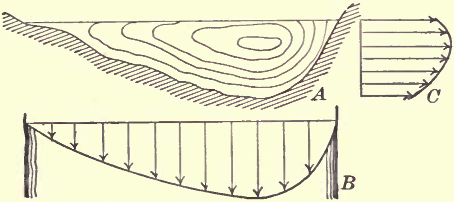


Fig. 68

a short distance below the surface. At B is shown a plan of the stream with arrows roughly representing the surface velocities; the greatest of these is seen to be near the deepest part of the channel, while the others diminish toward the banks, the curve showing the law of variation resembling a parabola. At C is shown by arrows the variation of velocities in a vertical line, the smallest being at the bottom and the largest a short distance below the surface; concerning this curve there has been much contention, but it is commonly thought to be a parabola whose axis is horizontal. These are the general laws of the variation of velocity throughout the cross-section; the particular relations are of a complex character, and vary so greatly in channels of different kinds that it is difficult to formulate them The ratio of the mean velocity to the maximum surface velocity in a river is usually about 0.80. The ratio of the mean velocity in any vertical to the surface velocity in that vertical is usually about 0.86. By measuring the velocity at the mid-depth in any vertical a very close approximation to the mean velocity in that vertical is obtained. A wind blowing upstream decreases the surface velocities and one blowing down stream increases them, without materially affecting the mean velocity.

Prob. 60 *A*. A stream 60 feet wide is divided into three sections, having the areas 32, 65, and 38 square feet, and the surface velocities near the middle of these sections are found to be 1.3, 2.6, and 1.4 feet per second. Compute the approximate discharge and mean velocity.

Prob. 60 *B*. When there is a bend in a stream, at what part of the cross-section is the water the deepest?

Prob. 60 *C*. What should be the length of a waste weir in order to carry one inch of rainfall per hour in a watershed of 3 square miles, the head on the crest of the weir being 2.5 feet?

CHAPTER 7

HYDRAULIC MOTORS

ART. 61. EFFICIENCY OF A MOTOR

A hydraulic motor is an apparatus for utilizing the energy of a waterfall. It generally consists of a wheel which is caused to revolve either by the weight of water falling from a higher to a lower level, or by the dynamic pressure due to the change in direction and velocity of a moving stream. When the water enters at only one part of the circumference, the apparatus is called a water wheel; when it enters around the entire circumference, it is called a turbine. The efficiency of a motor is the ratio of the work actually utilized to the theoretic energy. When the efficiency e is unity all of the theoretic energy is utilized, when e is zero none is utilized. The efficiency of hydraulic motors may range from 0.25 to 0.95; that is, the work actually performed by them may range from 25 to 95 percent of the theoretic energy of the waterfall.

When a weight of water W falls in each second through the height h, or when it is delivered with the velocity v, its theoretic energy per second is

$$K = Wh \qquad \text{or} \qquad K = Wv^2/2g$$

The actual work per second equals the theoretic energy minus all the losses of energy. These losses may be divided into two classes: first, those caused by the transformation of energy into heat; and second, those due to the velocity v_1 with which the water leaves the wheel. The first class includes losses in friction, and losses in foam and eddies. Let the loss of work due to this be

Wh', in which h' is the head lost by these causes. The second loss is due merely to the fact that the departing water carries away the energy $W.v_1^2/2g$. The work per second imparted by the water to the wheel then is

$$k = W(h - h' - v_1^2/2g)$$

and dividing this by the theoretic energy, the efficiency is,

$$e = 1 - h'/h - v_1^2/2gh$$

This formula is the basis of all discussions on the theory of water wheels and motors. It shows that e can only become unity when $h' = 0$ and $v_1 = 0$, and accordingly the two following fundamental conditions must be fulfilled in order to secure high efficiency:

1. The water must enter and pass through the wheel without losing energy in friction and foam.
2. The water must reach the level of the tail race without absolute velocity.

These two requirements are expressed in popular language by the well-known maxim "the water should enter the wheel without shock and leave without velocity." Here the word "shock" means that method of introducing the water upon the wheel which produces foam and eddies. It is the constant aim of designers to so arrange wheels as to avoid these losses and thus render the efficiency as high as possible.

Prob. 61. A wheel using 70 cubic feet of water per minute under a head of 12.4 feet has an efficiency of 63 percent. What effective horse-power does it deliver?

ART. 62. OVERSHOT WHEELS

In the overshot wheel the water acts largely by its weight. Fig. 69 shows an end view or vertical section, where h is the total fall from the surface of the water in the head race or flume to the surface in the tailrace. This total fall may be divided into three parts: that in which the water is filling the buckets, that in which the water is descending in the filled buckets, and that which remains after the buckets are emptied. Let the first of these parts be called h_0, and the last h_1. In falling the

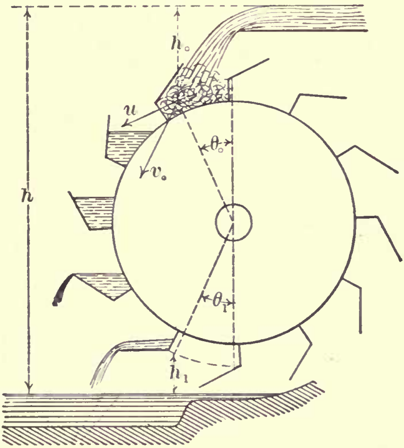


Fig. 69

distance h_0 the water acquires a velocity v_0 which is approximately equal to $\sqrt{2gh_0}$, and then, striking the buckets, this is reduced to u, the tangential velocity of the wheel, whereby a loss of energy in impact occurs. It then descends through the distance $h-h_0-h_1$, acting by its weight alone, and finally, dropping out of the buckets, reaches the level of the tail race with a velocity which causes a second loss of energy. Let h' be the head lost in entering the buckets, and let v_1 be the velocity $\sqrt{2gh^1}$ with which the water reaches the level of the tail race. Then the hydraulic efficiency of the wheel is given by the general formula in the last article. It may be shown, as in Treatise on Hydraulics, that when the wheel revolves at the most advantageous velocity h' equals $\frac{1}{2}h_0$. Then

$$e = 1 - \tfrac{1}{2}h_0/h - h_1/h$$

is the maximum efficiency of the overshot wheel.

This investigation shows that one-half of the entrance fall h_0 and the whole of the exit fall h_1 are lost, and it is hence plain that in order to make e as large as possible both h_0 and h_1 should be as small as possible. The fall h_0 is made small by making the radius of the wheel large, but it cannot be made zero, for then no water would enter the wheel; it is generally taken so as to make the angle θ_0 about 10 to 15 degrees. The fall h_1 is made small by giving to the buckets a form which will retain the water as long as possible. The practical advantageous velocity of the overshot wheel is found to be about $0.4v_0$, and its efficiency is found to be high, ranging from 70 to 90 percent. In times of drought, when the water supply is low, and it is desirable to utilize all the power available, its efficiency is the highest, since then the buckets are but partly filled and h_1 becomes small. Herein lies the great advantage of the overshot wheel; its disadvantage is in its large size and the expense of construction and maintenance.

Prob. 62. Estimate the horse-power and efficiency of an overshot wheel which uses 1080 cubic feet of water per minute under a head of 26 feet, the diameter of the wheel being 23 feet, and the water entering 15° from the top and leaving 12° from the bottom.

ART. 63. BREAST AND UNDERSHOT WHEELS

The breast wheel is applicable to small falls, and the action of the water is partly by impulse and partly by weight. As represented in Fig. 70, water from a reservoir is admitted through an orifice upon the wheel under the head h_0 with the velocity v_0; the water being then confined between the vanes and the curved breast acts by its weight through a distance h_2, until it is released at the level of the tail race and departs with the velocity u,

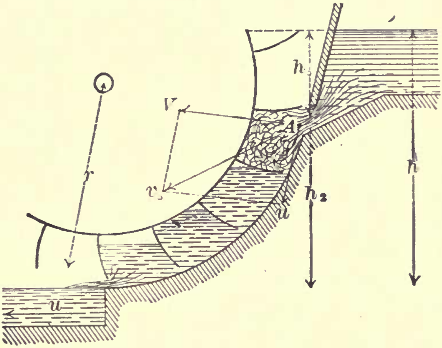


Fig. 70

which is the same as that of the circumference of the wheel. The total energy of the water being Wh, the work of the wheel is eWh, if e be its efficiency. Owing to leakage and to the larger loss in impact, the efficiency of the breast wheel is materially less than that of the overshot, and usually ranges from 50 to 80 percent.

The common undershot wheel has plane radial vanes and is set in a flowing stream so that the water enters and leaves almost in a horizontal direction. When v is the velocity of the entering water and u that of the circumference of the wheel the dynamic pressure developed (Art. 18) is $W(v-u)/g$ and the work done per second is $Wu(v-u)g$. This expression has its greatest value when $u = \frac{1}{2}v$, so that the maximum work is $\frac{1}{2}Wv^2/2g$ and the greatest efficiency is $\frac{1}{2}$. Experiments show that the advantageous velocity is about $0.4v$ instead of $0.5v$, and that the efficiency ranges from 0.25 to 0.40.

Prob. 63. Estimate the horse-power that can be obtained from an undershot wheel with plane radial vanes placed in a stream having a mean velocity of 5 feet per second, the width of the wheel being 15 feet, its diameter 8 feet, and the maximum immersion of the vanes being 1.33 feet.

ART. 64. VERTICAL IMPULSE WHEELS

A vertical wheel having small vanes against which the water is delivered from a nozzle, is often called an impulse wheel, or a "hurdy-gurdy" wheel. The Pelton wheel, the Cascade wheel, and other forms can be purchased in several sizes and are convenient on account of their portability. Fig. 71 shows an outline sketch of such a wheel with the vanes somewhat exaggerated in size. The simplest vanes are radial planes as at A, but these give a low efficiency. Curved vanes, as at B, are generally used, as these cause the water to turn backward, opposite to the direction of the motion, and thus to leave the wheel with a low absolute velocity. In the plan of the wheel it is seen that the vanes may be arranged so as also to turn the water sidewise while deflecting it backward. The experiments of Browne show that with plane radial vanes the highest efficiency was 40.2 percent, while with curved vanes or cups 82.5 percent was attained. The velocity of the vanes which gave the highest efficiency was almost exactly one-half the velocity of the jet.

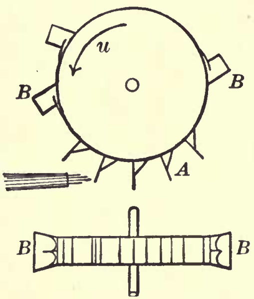


Fig. 71

The Pelton wheel is used under high heads, and hence it has a high velocity. These wheels are wholly of iron, and are provided with a casing to prevent the spattering of the water. Fig. 72 shows a form with three nozzles, by which three streams are applied at different parts of

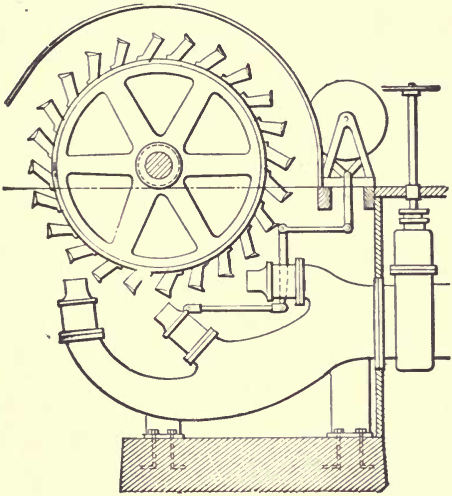


Fig. 72

the circumference, in order to obtain a greater power than by a single nozzle.

Prob. 64. The impinging jet on a hurdy-gurdy wheel has a diameter of 0.182 feet and a velocity of 58.5 feet per second. The efficiency of the wheel being 44.5 percent, what effective horsepower does it furnish?

ART. 65. THE REACTION WHEEL

The reaction wheel, invented by Barker about 1740, consists of a number of hollow arms connected with a hollow vertical shaft, as shown in Fig. 73. The water issues from the ends of the arms in a direction opposite to that of their motion, and by the dynamic pressure due to its reaction the energy of the water is transformed into useful work. Let the head of water CC in the shaft be h; then the pressure-head BB which causes the flow from the arms is greater than h, on account of the centrifugal force due to the rotation of the wheel. Let u_1 be the absolute velocity of the exit orifices, then BB is equal to $h+u_1^2/2g$, and the velocity of discharge relative to the wheel is

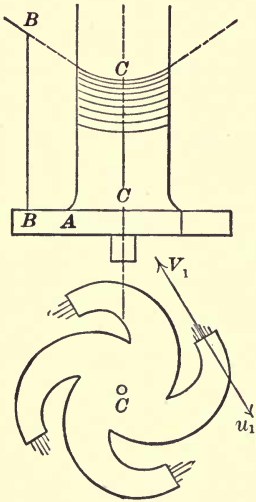


Fig. 73

$$V_1 = \sqrt{2gh+u_1^2}$$

The absolute velocity v_1 of the issuing water now is

$$v_1 = V_1 - u_1 = \sqrt{2gh+u_1^2} - u_1$$

and the theoretic efficiency of the wheel is

$$c = 1 - \frac{v_1^2}{2gh} = \frac{2u_1}{V_1+u_1}$$

This investigation shows that the efficiency of a reaction wheel increases with its speed and can only become unity when $u_1 = V_1$. Nothing approaching this can, however, be realized and, on account of losses due to friction, a very high speed in impracticable. When $V_1 = 3u_1$, the efficiency is only 0.50. The reaction wheel is not now used as a hydraulic motor, but it is of interest as being the starting point for the subject of turbines.

Prob. 65. Show that v_1^2 equals $(V_1-u_1)^2$, that $2gh$ equals $V_1^2-u_1^2$, that $v_1^2/2g$ equals $(V_1-u_1)/(V_1+u_1)$, and that e equals $2u_1/(V_1+u_1)$.

ART. 66. OUTWARD-FLOW TURBINES

A reaction turbine is driven by the dynamic pressure and reaction of flowing water (Art. 18) which at the same time is under a certain degree of static pressure. If in the reaction wheel of Fig. 73 the arms be separated from the penstock at A, and be so arranged that BA revolves around the axis while AC is stationary, the resulting apparatus may be called a reaction turbine. The static pressure of the head CC can still be transmitted through the arms, so that, as in the reaction wheel, the discharge is influenced by the speed. Fig. 74 gives a vertical

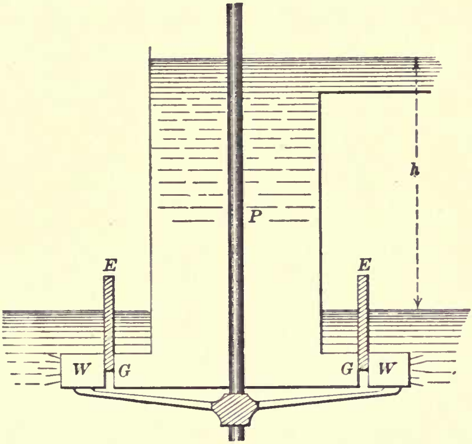


Fig. 74

section of an outward flow wheel W, to which water is brought by guides G from a fixed penstock P. Between the guides and the wheel there is a space in which slides an annular vertical gate E; this gate serves to regulate the quantity of water and the wheel stops when it is entirely depressed. Fig. 75 gives again the lower part of Fig. 74, and above it is a horizontal section showing the arrangement of the guides G and the buckets of the wheel W. The guides are curved so as better to direct the water against the buckets, and the buckets are curved so that the water may leave the wheel with low absolute velocity v_1. Water enters all around the circumference so that the buckets are fully filled. The wheel is attached by arms to the vertical shaft the motion of which delivers power in a building above.

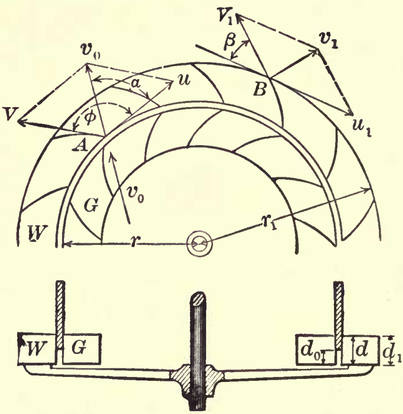


Fig. 75

The outward-flow turbine is often called the Fourneyron turbine, it having been invented by him in 1827. The guides and buckets with the gates and the surrounding casings, are made of iron. Numerous forms with different kinds of gates and different proportions of guides and vanes are in the market. They are made of all sizes from 6 to 60 inches in diameter, and larger sizes are built for special cases. The great turbines installed at Niagara in 1896 are of the outward-flow type, the inner diameter of a wheel being 63 inches and each twin turbine furnishing about 5000 horse-powers. The efficiency of turbines is greatest at full gate, ranging generally from 70 to 90 percent. There is a certain velocity, called the advantageous speed, with which the turbine must revolve in order to give the maximum efficiency.

Prob. 66. If the efficiency of a turbine is 75 percent when delivering 5000 horse-powers under a head of 136 feet, how many cubic feet of water per minute pass through it?

ART. 67. INWARD-FLOW TURBINES

The smaller sizes of turbines used in the United States are mostly of the inward-flow type, or of a combined inward- and downward-flow type. Fig. 76 shows an inward flow turbine set in a wooden penstock, from which the water enters the guides, then passes through the wheel, and is finally discharged downward through a draft tube. Fig. 77 shows horizontal vertical sections of this turbine,

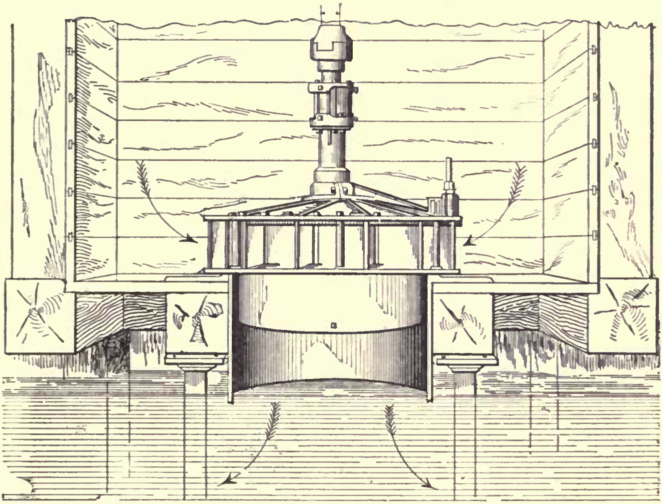


Fig. 76

G being the fixed guides and *W* the movable wheel, with an annular gate sliding vertically between them. The wheel is attached by arms to the vertical shaft which is supported in a step bearing below and carries the power into a building above. An inward-flow turbine is often called the Francis turbine, it having been invented by James B. Francis about 1850.

For both outward-flow and inward-flow turbines there is a certain advantageous speed u_1 at which the wheel must move in order to obtain the maximum power, while the depths d_0, d, d_1 and the angles α, ϕ, β in Figs. 75 and 77 must also be related by certain conditions. See Merriman's Treatise on Hydraulics for this theory.

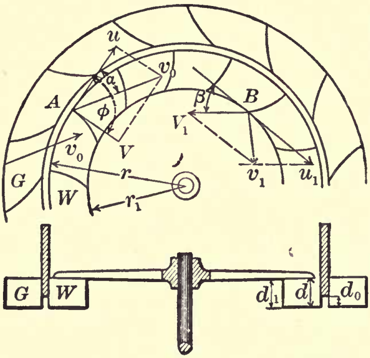


Fig. 77

Prob. 67. A test of an electric generator at Niagara showed that 5498 kilowatts were generated by a discharge of 447.2 cubic feet of water per second through the turbine under a head of 135.1 feet. The efficiency of the generator being 97 percent, what was the efficiency of the turbine?

ART. 68. MEASUREMENT OF POWER

The usual method of measuring the effective work of a hydraulic motor is by means of the friction brake or power dynamometer invented by Prony about 1780. In Fig. 78 is illustrated a simple method of applying the apparatus to a vertical shaft, the upper diagram being a plan and the lower an elevation. Upon the vertical shaft is a fixed pulley, and against this are seen two rectangular pieces of wood hollowed so as to fit it, and connected by two bolts. By turning the nuts on these bolts while the pulley is revolving, the friction can be increased at pleasure, even so as to stop the motion; around these bolts between the blocks are two spiral springs (not shown in the diagram) which press the blocks outward when the nuts are loosened. To one of these blocks is attached a cord which runs horizontally to a small movable pulley over which it passes, and supports a scale-pan in which weights are placed. This cord runs in a direction opposite to the motion of the shaft, so that when the brake is tightened, it is prevented from revolving by the tension caused by the weights. The direction of the cord in the horizontal plane must be such that the perpendicular let fall upon it from the center of the shaft, or its lever-arm, is constant; this can be effected by keeping the small pointer on the brake at a fixed mark established for that purpose.

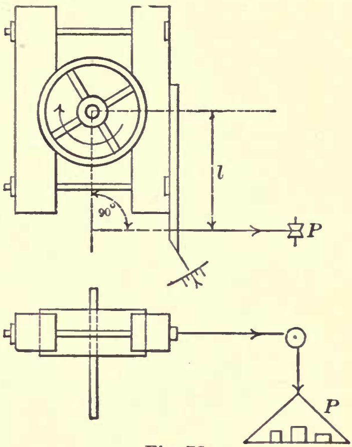


Fig. 78

To measure the work done by the wheel, the shaft is disconnected from the machinery which it usually runs and allowed to revolve, transforming all its work into heat by the friction between the revolving pulley and the brake, which is kept stationary by tightening the nuts and at the same time placing sufficient weights in the scale-pan to hold the pointer at the fixed mark. Let N be the number of revolutions per minute as determined by a counter attached to the shaft, P the tension in the cord, which is equal to the weight of the scale-pan and its loads, l the lever-arm of this tension with respect to the center of the shaft, r the radius of the pulley, and F the total force of friction between the pulley and the brake. Now in one revolution the force F is overcome through the distance $2\pi r$, and in N revolutions through the distance $2\pi rN$. Hence the effective work done by the wheel in one minute is

$$k = F \, . \, 2\pi rN = 2\pi N \, . \, Fr$$

The force F acting with the lever-arm r is exactly balanced by the load P acting with the lever-arm l; accordingly the moments Fr and Pl are equal, and hence the work done by the wheel in one minute is

$$k = 2\pi NPl$$

Lastly, if P is in pounds and l in feet, the formula for the effective horse-power of the wheel is

$$\overline{hp} = 2\pi NPl/33\,000$$

It is seen that this method is independent of the radius of the pulley, which may be of any convenient size; for a small motor the brake may be clamped directly upon the shaft, but for a large one a pulley of considerable size is needed and the brake is often made of iron and almost completely encircles the pulley. Both brake and pulley sometimes become hot, to prevent which a stream of cool water is allowed to flow upon them. The work or power measured is that delivered at circumference of the pulley, and does not include that power which is required to overcome the friction of the shaft upon its bearings. The shaft or axis of every water-wheel must have at least two bearings, the friction of which consumes probably about 3 percent of the power. The hydraulic power of the wheel, regarded as a user of water, is hence about 3 percent greater than that computed from above formula. The efficiency of the wheel is found by dividing the effective work by the theoretic work. Let W be the weight of water delivered per minute and h the head; then $e = k/Wh$, which is usually about 3 percent too large.

Prob. 68 *A*. Find the power and efficiency of a motor which makes 670 revolutions per minute, the weight on the brake being 2 pounds 14 ounces and its lever-arm 1.33 feet, and the theoretic power of the fall being 1.38 horse-power.

Prob. 68 *B*. Let $P = 12.5$ pounds, $l = 14.31$ feet, and $N = 635$; also let the discharge per second, as measured by a weir, be 4.81 cubic feet per second which is delivered upon the wheel under a head of 50.1 feet. Compute the effective power and efficiency.

Prob. 68 *C*. For a certain turbine the water from the tail race was measured over a weir with end contractions, the length of the weir being 1.909 feet and the head on the crest 0.287 feet. During the test the gage in the penstock read 11.25 feet and that in the tail race read 0.30 feet. The weight on the brake was 3.0 pounds and its lever arm was 1.431 feet. Compute the theoretic power, the effective power, and the efficiency of the motor.

CHAPTER 8

PUMPS AND PUMPING

ART. 69. THE SUCTION PUMP

The term "suction" is a misleading one unless it be clearly kept in mind that water will not rise in a vacuum tube unless the atmospheric pressure can act underneath it. For example, no amount of rarefaction above the surface of the water in a glass bottle will cause that water to rise. When the tube is inserted into a river or pond, however, the water will rise in it when a partial vacuum is formed, since the atmospheric pressure which is transmitted through the water pushes it up until equilibrium is secured. The mean atmospheric pressure of 14.7 pounds per square inch at the sea level is equivalent to a height of water of 34 feet, and this is the limit of raising water by suction alone. In practice this height cannot be reached on account of the impossibility of producing a perfect vacuum, and it is found that about 28 feet is the maximum height of suction lift.

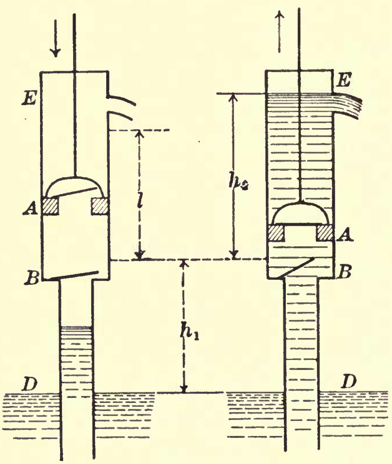


Fig. 79

Fig. 79 gives two diagrams illustrating the principle of action of the common suction and lift pump. It consists of two vertical tubes BD and BE, the former being called the suction pipe and the latter the pump cylinder. The piston A in the pump cylinder has a valve opening upward, and the valve B at the top of the suction pipe also opens upward. In the left-hand diagram the piston is descending, the valve A being open and B being closed under the pressure of the air in the space between them. In the right-hand diagram the piston is ascending, the valve A being closed by the pressure of the air or water above it, while B is open, owing to the excess of atmospheric pressure below it.

Let h_1 be the distance from the water level D to the lowest position of the piston; this is called the height of lift by suction. Let h_2 be the height from the lowest position of the piston to the spout where the water flows out; this is called the height of lift by the piston. The distance h_1+h_2 is the vertical height through which the water is raised, and if W be the weight of water raised in one second, the useful work per second is $W(h_1+h_2)$. The energy imparted to the pump through the piston rod is always greater than this useful work, since energy is required to overcome the frictional resistances. The action of this pump is intermittent, and water flows from the spout only during the upward stroke of the piston. When there are N upward strokes per minute, the discharge in one minute is NAl, if the piston and its valve are tight. The useful work per minute is $NwAl(h_1+h_2)$, if w is the weight of a cubic unit of water. When l and h_1+h_2 are in feet, A in square feet, and w in pounds per cubic foot, the horse-power expended in this useful work is

$$\overline{HP} = NwAl(h_1+h_2)/33{,}000$$

and to this must be added the horse-power required to overcome the resistances of friction and inertia. This additional power often amounts to as much as that needed for the useful work, and in this case the efficiency of the pump is 50 percent. Suction and lift pumps are of numerous styles and sizes, the simplest being of square wooden tubes or of round tin-plate tubes with leather valves, and these can be readily made by a carpenter or tinsmith.

Prob. 69. The diameter of the pump cylinder is 8 inches and that of the suction pipe is 6 inches, while the vertical distance from the water level to the spout is 23 feet. If the pump piston makes 30 upward strokes per minute, each 9 inches long, what horse-power is required to operate the pump if its efficiency is 45 percent?

ART. 70. THE FORCE PUMP

A force pump is one that has a solid piston which can transmit to the water the pressure exerted by the piston rod and thus cause it to rise in a pipe. The early force pumps had little or no suction lift, as the pump cylinder was immersed in the body of water which furnished the supply, but the modern forms usually operate both by suction and pressure, the former occurring in a suction pipe and the latter in the pump cylinder. Fig. 80 shows the principle of action of the common vertical single-acting suction and force pump in which there is no water above the

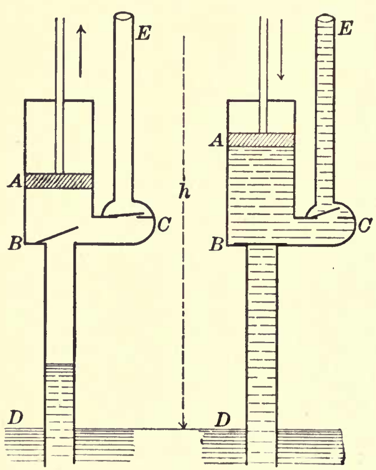


Fig. 80

piston. In the left-hand diagram the piston is ascending, and the water is rising in the suction pipe *BD* under the upward atmospheric pressure; this ascent of the water occurs in exactly the same manner as explained in Art. 69, and after several strokes its level rises above the suction valve *B*. The right-hand diagram shows the piston descending and forcing the water up the discharge pipe *CE*. At *C*, where this pipe joins the pump cylinder, is a check valve which closes on the upward stroke and thus prevents the water in *CE* from returning into the pump cylinder, while it opens on the downward stroke under the upward pressure of the water.

The cylinder of the single-acting pump may also be placed horizontally, the vertical suction and discharge pipes being connected to one end of the cylinder. The action is intermittent and hence not advantageous for large pumps. The double-acting pump is one having a

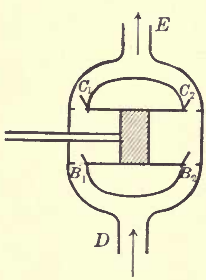


Fig. 81

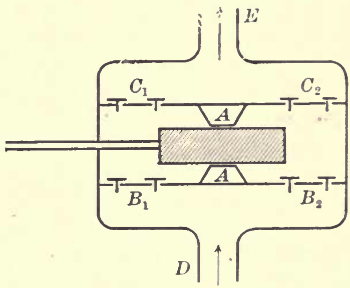


Fig. 82

single cylinder in which a solid piston or plunger exerts suction and pressure in both strokes and thus gives a nearly continuous flow through suction and discharge pipes. Fig. 81 shows the form known as the piston pump while Fig. 82 is that called the plunger pump, the piston being replaced by a long cylinder moving in a short stuffing box AA. In both figures D is the suction pipe and E the discharge pipe. When the piston moves from left to right, the valves B_1 and C_2 open, while B_2 and C_1 close; when it moves in the opposite direction, B_2 and C_1 open, while B_1 and C_2 close. The cylinder of the piston pump must be bored to an exact and uniform size, and its piston must be carefully packed, while in the plunger pump only the short length of the stuffing box is bored and packed, the plunger itself having no packing. The water lifted in one stroke is Al, where A is the area of the piston and l the length of its stroke, provided there is no leakage. The expression for the work utilized is the same as that given in Art. 69, but the efficiency is usually much greater, it being 80 or 90 percent in the best forms with low lifts and speeds.

Prob. 70. Consult Ewbanks' Hydraulics and Mechanics (New York, 1847), and describe a method of raising water through a low lift by means of a frictionless plunger pump. Ewbank notes that a stout young man weighing 134 pounds raised 8⅓ cubic feet per minute with this machine to a height of 11½ feet, and worked at this rate nine hours per day. If the efficiency of this pump was unity, what horse-power did the stout young man exert?

ART. 71. PUMPING ENGINES

The modern pumping engine consists of one or more steam cylinders connected to the same number of pump cylinders by piston rods, so that the steam pressure is directly transmitted through them to the water. The water cylinders are usually of the plunger type, and these are connected to the same suction and discharge pipes, an air chamber being placed on the latter to relieve the pump chambers of shock and to insure steady flow.

The term "duty" is often employed as a measure of the performance of a pumping engine, instead of express-ing it by an efficiency percentage. This term was devised by Watt, who defined duty as the number of foot-pounds of useful work produced by the consumption of 100 pounds of coal. A more precise definition of duty was introduced in 1890 by the American Society of Mechanical Engineers, namely, that duty should mean the number of foot-pounds of work produced by the expenditure of 1 000 000 British thermal heat units. One British thermal heat unit is that amount of energy which will raise one pound of pure water one Fahrenheit degree in temperature when the water is at or near the temperature of maximum density (Art. 3); this amount of energy is 778 foot-pounds, and this constant is called the mechanical equivalent of heat. The duty of a perfect pumping engine, in which no losses of any kind occur, would be 778 000 000 foot-pounds. The highest duty obtained in a test is about 180 000 000 foot-pounds, and the efficiency of such an engine is $180/778 = 0.23$. Common pumping engines have duties ranging from 120 000 000 to 60 000 000, the corresponding efficiencies being from 15 to 7.5 percent. The modern definition of duty agrees with that of Watt, if the coal is of such quality that one pound of it possesses a potential energy of 10 000 British heat units, which is somewhat less than that obtainable from average coal. The higher the duty of a pumping engine the greater is the amount of work that can be performed by burning a given quantity of coal. A high-duty engine is hence economical and a low-duty engine is wasteful in coal consumption, but the first cost of the former is much greater than that of the latter.

Prob. 71. A certain pumping engine had a water plunger of 172 square inches section area, the length of stroke being 18.9 inches. During a test of 12 hours the number of single strokes was 76 000, and the leakage past the plunger was 5900 cubic feet. How many gallons per minute were discharged by the pump?

ART. 72. THE CENTRIFUGAL PUMP

The centrifugal pump is the reverse of a turbine wheel, and any reaction turbine, when run backwards by power applied to its axle, will raise water through its penstock. Fig. 83 shows the principle of the arrangement and action

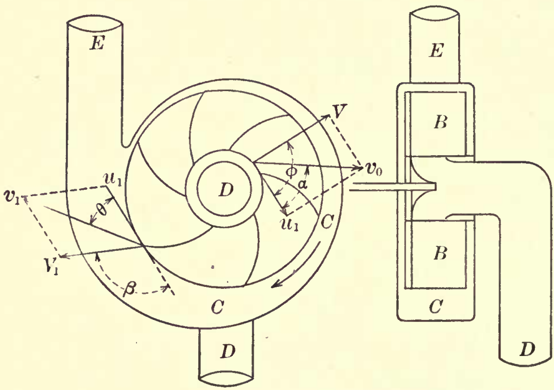


Fig. 83

of the centrifugal pump. The power is applied through the axis to rotate the wheel *BB* in the direction indicated by the arrow. This wheel is formed of a number of curved vanes like those in a turbine wheel (Art. 66). The revolving vanes produce a partial vacuum, and this causes the water to rise in the suction pipe *DD* which enters through the center of the wheel case and delivers the water at the axis of the wheel. The water is then forced outward through the vanes and passes into the volute chamber *CC*, which is of varying cross-section in order to accommodate the increasing quantity of water that is delivered into it, and all of which passes up the discharge pipe *E*. The rotation of the wheel hence produces a negative pressure at the upper end of the suction pipe and a positive pressure in the volute chamber, and the water rises in the pipes in the same manner as in those of a suction and force pump. The height of the suction lift cannot usually exceed about 28 feet.

The centrifugal pump possesses an advantage over the force pump in having no valves and in being able to handle muddy water, for even gravel may pass through the vanes without injuring them. The above figures represent the principle rather than the actual details of construction. Usually the suction pipe is divided into two parts which enter the axis upon opposite sides of the wheel, and the volute chamber is often made wider than the wheel case, thus forming what is called a whirlpool chamber, which prevents some of the losses of head due to impact. The efficiency of the pump usually ranges from 40 to 60 percent, it being the highest for low lifts.

Prob. 72. A centrifugal pump lifts 120 cubic feet of water per minute through a height of 18 feet, the power required to do this being 8.2 horse-powers. Compute the efficiency of the pump and the head lost in friction.

ART. 73. COMPRESSED-AIR PUMPS

Pumps which have no moving parts and which operate through the action of air suction and dynamic pressure constitute another class. Here belong the jet or ejector pumps which act largely through suction, and the injector pump used on locomotives. The latter produces a vacuum through the flow of steam, and cannot be discussed here, as it involves principles of thermodynamics. The fundamental principle, however, is indicated in Fig. 84, which shows the jet apparatus invented by James Thomson in 1850. The water to be lifted is at C, and it rises by suction to the chamber B, from which it passes through the discharge pipe to the tank *D*. The forces of suction and pressure are produced by a jet of water issuing from a nozzle at the mouth of the discharge pipe, the nozzle being at the end of a pipe *AB* through which water is brought from a reservoir; or the water delivered from the nozzle may come from a hydrant or force pump.

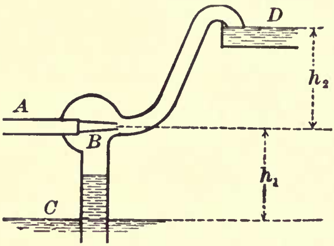


Fig. 84

The air lift pump is extensively used for raising water from deep wells, compressed air which is forced down a vertical pipe in the well tube issuing from its lower end. As it issues, bubbles are formed in the entire column of water in the well tube, and being lighter than a column of common water, it rises to a greater height under the atmospheric pressure. In this manner water having a natural level 50 feet or more below the surface of the ground may be caused to rise above that surface.

Among the many forms of pumps operating under the pressure of compressed air only the ejector pump used in the Shone system of sewerage can here be mentioned. The sewage from a number of houses flows to a closed basin, called an injector, in which it continues to accumulate until a valve is opened by a float. The opening of this valve allows compressed air to enter, and this drives out the sewage through a discharge pipe to the place where it is desired to deliver it.

Prob. 73. Consult engineering journals of 1893 and ascertain facts about the Shone ejectors used at the World's Columbian Exposition in Chicago.

ART. 74. THE HYDRAULIC RAM

The hydraulic ram is an apparatus which employs the dynamic pressure produced by stopping a column of moving water to raise a part of this water to a higher level than that of its source. The principle of the action of the hydraulic ram is shown in Fig. 85, where *A* is the

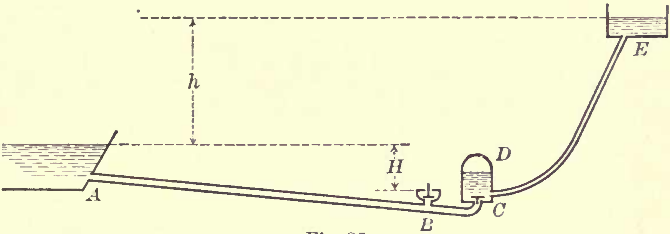


Fig. 85

reservoir that furnishes the supply, *BCD* the ram, *AB* the drive pipe which carries the water to the ram, *DE* the discharge pipe through which a part of the water is raised to the tank *E*. The ram itself consists merely of the waste valve *B* through which a part of the water from the drive pipe escapes, and the air vessel *D* which has a valve *C* that allows water to enter it through *BC*, but prevents its return. The waste valve *B* is either weighted or arranged with a spring so that it will open when acted upon by the static pressure due to the head *H*. As soon as it opens the water flows through it, but as the velocity increases the dynamic pressure or impulse (Art. 18) due to the motion of the column *AB* becomes sufficiently great to close the valve *B*. Then this dynamic pressure opens the valve *C* and compresses the air in the air chamber or forces water up the discharge pipe. A moment later when equilibrium has obtained in the air vessel, the valve *C* closes and the air pressure maintains the flow for a short period in the discharge pipe, while the water in the drive pipe comes to rest. Then the waste valve B opens again, and the same operations are repeated.

The least possible fall in the drive pipe of the hydraulic ram is about $1\frac{1}{2}$ feet and the least length of drive pipe about 15 feet. It is customary to make the area of the discharge pipe from one-third to one-fourth that of the drive pipe, and with these proportions a fall of 10 feet will force water to a height of nearly 150 feet. A common rule of manufacturers is that about one-seventh of the water flowing down the drive pipe may be raised to a height five times that of the fall in the drive pipe; this is a rough rule only, for the length of the discharge pipe is one of the controlling factors as well as its vertical rise.

Prob. 74. Consult Ewbanks' Mechanics and Hydraulics (New York, 1847) and ascertain the circumstances which led to the invention of the hydraulic ram.

ART. 75. PUMPING THROUGH PIPES

A pump is often used to force water directly through the mains of a water-supply system under a designated pressure. The work of the pump in this case consists of that required to maintain the pressure and that required to overcome the frictional resistances. To reduce the injurious resistance to the smallest limits the mains should be large in order that the velocity of flow may be small. In Fig. 86 is shown a symbolic representation of the case of pumping into a main, P being the pump, C the source of supply, and DM the pressure-head which is maintained upon the end of the pipe during the flow. At the pump the pressure-head is AP, so that AD represents the hydraulic gradient (Art. 38) for the pipe from P to M. The total work of the pump may then be regarded as expended in lifting the water from C to A, and this consists of three parts corresponding to the heads CM or z, MD or h_1, and AB; the first overcoming the force of gravity, the second maintaining the discharge under the required pressure, while the last is transformed into heat in overcoming friction and other resistances. In this direct method of water supply a standpipe AP is often erected near the pump, in which the water rises to a height corresponding to the required pressure, and which furnishes a small supply when a temporary stoppage of the pumping engine occurs.

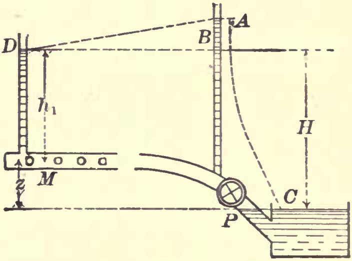


Fig. 86

For example, let it be required to find the horse-power of a pump to raise 1 200 000 gallons per day through a height of 230 feet in a pipe of 6 inches diameter and 1400 feet length. These data give the discharge per second as 1.86 cubic feet and the velocity in the pipe as 9.47 feet per second, which corresponds to a velocity-head of 1.39 feet. The probable head lost in entering the pipe (Art. 33) is $0.5 \times 1.39 = 0.7$ feet. For a new clean pipe the friction factor f (Art. 34) is about 0.020, whence the loss of head in friction is about 77.8 feet. The other losses of head depend upon the details of the pump cylinder and valves and may be taken as 4 times the velocity-head or 5.6 feet. The total head to be overcome is $230 + 0.7 + 77.8 + 5.6 = 314.1$ feet. The work to be performed per second then is $62.5 \times 1.86 \times 314.1 = 36\,510$ foot pounds, and the horse-power to be expended is $36\,510/550 = 66.4$. Here the useful head is 230 feet while the total head is

314.1 feet, so that the efficiency of the pump and pipe is only 314/230 = 73 percent. The losses can be much diminished by using a larger pipe.

Prob. 75 *A*. For the above data let the 6-inch pipe be replaced by one 14 inches in diameter. Compute the velocity in the pipe, and the horse-power required.

Prob. 75 *B*. A house is 60 feet lower than a spring *A* and 30 feet higher than a spring *B*. A pipe from *A* to the house runs near *B*. Explain a method by which the water from *B* can be drawn into the pipe and be delivered at the house.

Prob. 75 *C*. What is the efficiency of a bucket pump which lifts 2000 liters of water per minute through a height of 3.5 meters with an expenditure of 2.5 metric horse-powers?

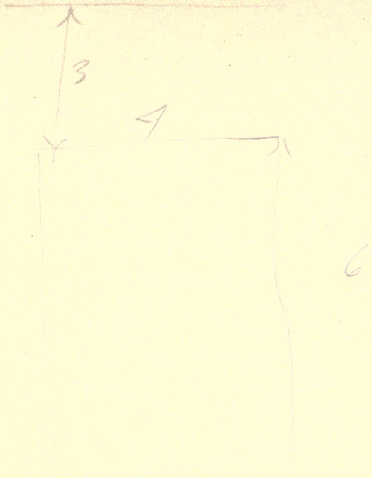
Prob. 75 *D*. The calorie is the metric thermal unit, this being the energy required to raise one kilogram of water one degree centigrade when the temperature of the water is near that of maximum density. How many calories are equivalent to 1 000 000 British thermal units?

ART. 76. ANSWERS TO PROBLEMS

Below will be found answers to some of the problems given in the preceding pages, the numbers of the problems being placed in parentheses. In general it is not a good plan for a student to solve a problem in order to obtain a given answer. One object of solving problems is, of course, to obtain correct results, but the correctness of those results should be established by methods of verification rather than by the authority of a given answer. However satisfactory it may be to know in advance the result of the solution of an exercise, let the student bear in mind that after commencement day answers to problems will not be given.

(1 *B*) One horse-power.
(3) 147.2 pounds.
(4 *A*) 960 feet.
(6 *A*) 676.9 net tons per square foot.
(8) 2719 pounds.

(9 C) 10.26 feet.
(11 C) 3 horse-powers.
(11 F) 45.6 pounds.
(12 B) 56.9 feet per second.
(13 B) 12.9 feet per second.
(16 B) 32.1 feet per second.
(18 B) 19.3 pounds.
(20 B) 0.71 cubic feet per second.
(20 E) 66.7 feet per second.
(21 A) 0.0326 cubic feet per second.
(24 A) 120 gallons per minute.
(25 B) 0.602.
(29 A) 0.802.
(31 A) 0.28 feet.
(32 B) 24 feet.
(32 E) 79.6 feet per second.
(33 A) 0.29 feet.
(37 A) 3.538 feet.
(40 A) 18.4 feet.
(42 A) 11.3 feet.
(43 A) 1.4 feet.
(45) 6.0 feet.
(46 A) 233/283.
(47) $c=123$.
(51 B) 364 pounds.
(59) 3.9 feet per second.
(62) From 48 to 50 horse-power.
(63) From 12 to 14 horse-power.
(68 C) $e=0.35$.
(70) About one-sixth of a horse-power.
(72) $e=$about 50 percent.
(75 A) 48.9 horse-powers.



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